

Last Name: SOLUTIONS

First Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

## Quiz 8 (A)

**Question 1.** (3 marks) Write the vector  $\vec{u} = (1, 2, 4)$  as a sum of two vectors, one that is parallel to  $\vec{v} = (0, 4, 3)$  and one that is perpendicular to  $\vec{v}$ .

$$\vec{w}_1 = \text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v} = \frac{(1, 2, 4) \cdot (0, 4, 3)}{(0, 4, 3) \cdot (0, 4, 3)} (0, 4, 3) = \frac{20}{25} (0, 4, 3) = (0, \frac{16}{5}, \frac{12}{5})$$

$$\vec{w}_2 = \vec{u} - \vec{w}_1 = (1, 2, 4) - (0, \frac{16}{5}, \frac{12}{5}) = (1, -\frac{6}{5}, \frac{8}{5})$$

$$\vec{u} = \vec{w}_1 + \vec{w}_2 = (0, \frac{16}{5}, \frac{12}{5}) + (1, -\frac{6}{5}, \frac{8}{5})$$

PARALLEL TO  $\vec{v}$                       PERPENDICULAR TO  $\vec{v}$

**Question 2.** (3 marks) Given  $\vec{p} = (5, k)$ , and  $\vec{v} = (3, 7)$ . Find  $k$  such that

(a)  $\vec{p}$  and  $\vec{q}$  are parallel(b)  $\vec{p}$  and  $\vec{q}$  are orthogonal

$$a) \vec{p} = l \vec{q} \Rightarrow (5, k) = l(3, 7)$$

$$5 = 3l \Rightarrow l = \frac{5}{3}$$

$$\therefore k = 7l = 7\left(\frac{5}{3}\right) = \frac{35}{3}$$

$$b) \vec{p} \cdot \vec{q} = 0$$

$$(5, k) \cdot (3, 7) = 15 + 7k = 0$$

$$7k = -15$$

$$k = -\frac{15}{7}$$

**Question 3.** (4 marks) Find the volume of the parallelepiped determined by the vectors  $\vec{u} = (2, -1, -3)$ , and  $\vec{v} = (4, 1, -3)$  and  $\vec{w} = (2, -1, 4)$ .

$$\text{VOLUME} = \left| \vec{u} \cdot (\vec{v} \times \vec{w}) \right| = \begin{vmatrix} 2 & -1 & -3 \\ 4 & 1 & -3 \\ 2 & -1 & 4 \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ 4 & 1 \\ 2 & -1 \end{vmatrix}$$

$$= | 8 + 6 + 12 + 6 - 6 + 16 | = | 42 |$$

$$= 42$$