April 1, 2010

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 8 (B)

Question 1. *(3 marks)* Write the vector $\vec{u} = (3,1,2)$ as a sum of two vectors, one that is parallel to $\vec{v} = (0,3,4)$ and one that is perpendicular to $\vec{v}$.

$$\vec{w}_1 = \text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{v} = \frac{(3,1,2)\cdot(0,3,4)}{(0,3,4)\cdot(0,3,4)}(0,3,4) = \frac{11}{25}(0,3,4) = \left(0, \frac{33}{25}, \frac{44}{25}\right)$$

$$\vec{w}_2 = \vec{u} - \vec{w}_1 = (3,1,2) - \left(0, \frac{33}{25}, \frac{44}{25}\right) = \left(3, -\frac{8}{25}, \frac{6}{25}\right)$$

PARALLEL TO $\vec{v}$ PERPENDICULAR TO $\vec{v}$

$$\therefore \vec{u} = \vec{w}_1 + \vec{w}_2 = \left(0, \frac{33}{25}, \frac{44}{25}\right) + \left(3, -\frac{8}{25}, \frac{6}{25}\right)$$

Question 2. *(3 marks)* Given $\vec{p} = (3,k)$, and $\vec{q} = (7,5)$. Find k such that

(a) $\vec{p}$ and $\vec{q}$ are parallel

(b) $\vec{p}$ and $\vec{q}$ are orthogonal

a) $\vec{p} = l\vec{q} \Rightarrow (3,k) = l(7,5)$

$\therefore 3 = 7l \Rightarrow l = \frac{3}{7}$

$\therefore k = 5l = 5\left(\frac{3}{7}\right) = \frac{15}{7}$

b) $\vec{p}\cdot\vec{q} = 0$

$(3,k)\cdot(7,5) = 21 + 5k = 0$

$\therefore k = -\frac{21}{5}$

Question 3. *(4 marks)* Find the volume of the parallelepiped determined by the vectors $\vec{u} = (2,-2,-4)$, and $\vec{v} = (2,1,-1)$ and $\vec{w} = (5,-2,3)$.

$$\text{VOLUME} = |\vec{u}\cdot(\vec{v}\times\vec{w})| = \left|\begin{vmatrix} 2 & -2 & -4 \\ 2 & 1 & -1 \\ 5 & -2 & 3 \end{vmatrix}\right| \begin{matrix} 2 & -2 \\ 2 & 1 \\ 5 & -2 \end{matrix}$$

$$= |6 + 10 + 16 - (-20) - (4) - (-12)|$$

$$= 60$$