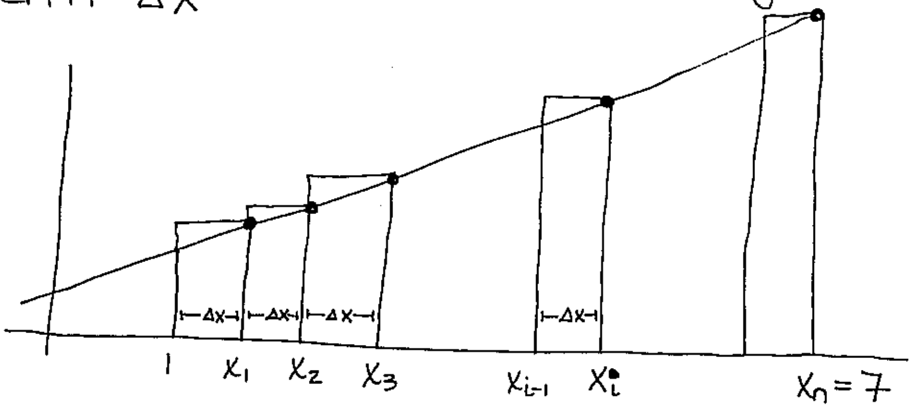
ASSIGNMENT #10
NYA ELECTROTECH
SOLUTIONS

[GOAL] COMPUTE AREA UNDER $f(x) = 3x+2$ between $1 \le x \le 7$

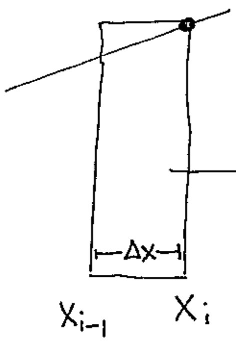
WE break up the Area INTO n rectangles EACH WITH width ΔX



NOTE:

$$X_1 = 1 + \Delta X$$
$$X_2 = 1 + 2\Delta X$$
$$X_3 = 1 + 3\Delta X$$
$$\vdots$$

IN general we say $X_i = 1 + i\Delta X$



$\rightarrow$ Area OF i^{th} rectangle

$$= \Delta X \cdot f(X_i)$$
$$= \Delta X \left(3(1+i\Delta X)+2\right)$$
$$= \Delta X \left(i\Delta X + 5\right)$$
$$= i(\Delta X)^2 + 5\Delta X$$

NOW NOTE THAT $\Delta X = \dfrac{6}{n}$ $\leftarrow$ length OF INTERVAL $\leftarrow$ divided into n pieces

So Area of ith rectangle $= 3i\left(\frac{6}{n}\right)^2 + 5\left(\frac{6}{n}\right)$

$$= \frac{108i}{n^2} + 5\left(\frac{6}{n}\right)$$

To get the area of all n rectangles we must add all the areas up:

$$\sum_{i=1}^{n} \frac{108i}{n^2} + 5\left(\frac{6}{n}\right)$$

$$= \underbrace{\frac{108}{n^2}}_{\text{A constant}} \sum_{i=1}^{n} i + \sum_{i=1}^{n} \underbrace{\frac{30}{n}}_{\text{also A constant}}$$

$$= \frac{108}{n^2} \frac{n(n+1)}{2} + \frac{30}{n}(n)$$

$$= \frac{54}{n^2}(n^2+n) + 30$$

$$= 54 + \frac{54}{n} + 30 = 84 + \frac{54}{n}$$

Now to get the actual area beneath the curve we consider an "infinite # of rectangles"

$$\lim_{n\to\infty} 84 + \frac{54}{n}$$

$$= 84$$

The area is 84 units^2