ASSIGNMENT #3
NYA ELECTROTECH
SOLUTIONS
WINTER 2010

1

$$\lim_{x\to 2} \frac{2-\sqrt{x+2}}{x-2}\,\frac{(2+\sqrt{x+2})}{(2+\sqrt{x+2})}$$

$$= \lim_{x\to 2} \frac{4-(x+2)}{(x-2)(2+\sqrt{x+2})}$$

$$= \lim_{x\to 2} \frac{2-x}{(x-2)(2+\sqrt{x+2})}$$

$$= \lim_{x\to 2} \frac{-(x-2)}{(x-2)(2+\sqrt{x+2})}$$

$$= \lim_{x\to 2} \frac{-1}{2+\sqrt{x+2}}$$

$$= \lim_{x\to 2} \frac{-1}{2+\sqrt{2+2}}$$

$$= \boxed{-\frac{1}{4}}$$

2

$$\lim_{x\to} \frac{(2x-1)^2-1}{2x-2}$$

$$= \lim_{x\to 1} \frac{4x^2-4x+1-1}{2x-2}$$

$$= \lim_{x\to 1} \frac{4x(x-1)}{2(x-1)}$$

$$= \lim_{x\to 1} 2x = \boxed{2}$$

3

$$\lim_{x\to\infty} \frac{x-1}{7x+4}$$

$$= \lim_{x\to\infty} \frac{\frac{x}{x}-\frac{1}{x}}{\frac{7x}{x}+\frac{4}{x}}$$

$$= \lim_{x\to\infty} \frac{1-\cancel{\frac{1}{x}}^{\,0}}{7+\cancel{\frac{4}{x}}^{\,0}}$$

$$= \boxed{\frac{1}{7}}$$

4

$$\lim_{x\to\infty} \frac{1-2x^2}{(4x+3)^2}$$

$$= \lim_{x\to\infty} \frac{1-2x^2}{16x^2+24x+9}$$

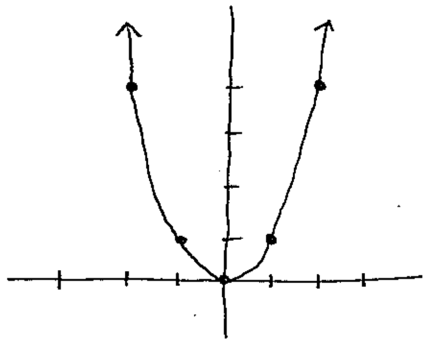
$$= \lim_{x\to\infty} \frac{\frac{1}{x^2}-\frac{2x^2}{x^2}}{\frac{16x^2}{x^2}+\frac{24x}{x^2}+\frac{9}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\cancel{\frac{1}{x^2}}^{\,0}-2}{16+\cancel{\frac{24}{x}}^{\,0}+\cancel{\frac{9}{x^2}}^{\,0}}$$

$$= -\frac{2}{16} = \boxed{-\frac{1}{8}}$$

5 $f(x) = \begin{cases} \dfrac{x^3 - x^2}{x-1} & \text{for } x \neq 1 \\ 1 & \text{for } x = 1 \end{cases}$

Simplify: $\dfrac{x^3 - x^2}{x-1} = \dfrac{x^2(x-1)}{(x-1)} = x^2$ so graph looks like $y = x^2$ except at $x = 1$



conditions for continuity at $x=1$

1. $f(1)$ exists
   $f(1) = 1$

2. $\lim\limits_{x \to 1^-} f(x)$
   $= \lim\limits_{x \to 1^+} f(x)$
   $= \lim\limits_{x \to 1} \dfrac{x^3 - x^2}{x-1}$
   $= \lim\limits_{x \to 1} \dfrac{x^2(x-1)}{(x-1)}$
   $= \lim\limits_{x \to 1} x^2 = 1$

   $\lim\limits_{x \to 1} f(x)$ exists & is equal to 1

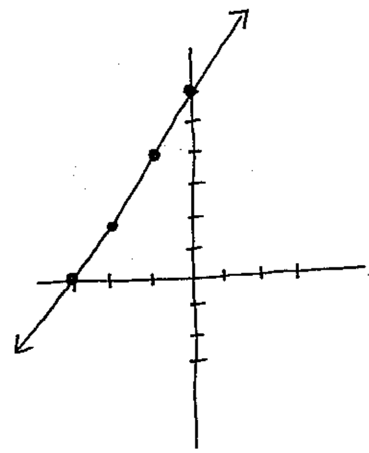
3. $f(1) = \lim\limits_{x \to 1} f(x)$

$f(x)$ $\boxed{\text{is continuous}}$

6. $$f(x) = \begin{cases} \dfrac{2x^2-18}{x-3} & \text{for } x<3 \text{ or } x>3 \\ 12 & \text{for } x=3 \end{cases}$$

Graph

$$\frac{2x^2-18}{x-3} = \frac{2(x^2-9)}{x-3} = \frac{2\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = 2(x+3)$$



Conditions for continuity

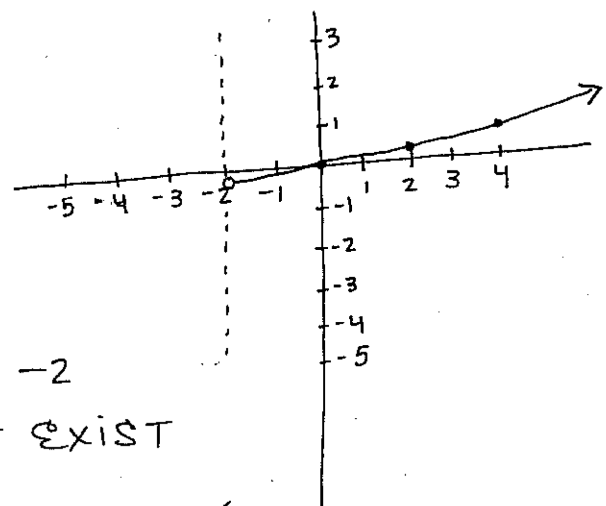
(1) $f(3)$ exists $f(3) = 12$

(2) $$\lim_{x\to3} f(x) = \lim_{x\to3} \frac{2x^2-18}{x-3}$$
$$= \lim_{x\to3} \frac{2(x+3)(x-3)}{(x-3)}$$
$$= \lim_{x\to3} 2(x+3) = 2(3+3) = 12$$

(3) $$f(3) = \lim_{x\to3} f(x)$$

$f(x)$ $\boxed{\text{is continuous}}$

7. $$f(x) = \begin{cases} \dfrac{x+2}{x^2-4} & x<-2 \\ x/8 & x>-2 \end{cases}$$



$f(x)$ is $\boxed{\text{NOT CONTINUOUS}}$ at -2
because $f(-2)$ does not exist

Graph $$\frac{x+2}{x^2-4} = \frac{\cancel{x+2}}{\cancel{(x+2)}(x-2)} = \frac{1}{x-2}$$