ASSIGNMENT #6 (SOLUTIONS) NYA ELECTROTECH

(9th edition)

P. 689

#12 $y = 6x - 2x^5$

$y' = 6 - 10x^4$

$\boxed{y'' = -40x^3}$

#14 $r = 3\theta^2 - \frac{20}{\sqrt{\theta}}$

$= 3\theta^2 - 20\theta^{-1/2}$

$r' = 6\theta + 10\theta^{-3/2}$

$\boxed{r'' = 6 - 15\theta^{-5/2}}$

#16

$f(x) = \sqrt[3]{6x+5}$

$= (6x+5)^{1/3}$

$f'(x) = \frac{1}{3}(6x+5)^{-2/3}$

$\boxed{f''(x) = -\frac{2}{9}(6x+5)^{-5/3}}$

#18 $f(x) = \frac{7.5}{\sqrt{3-4x}}$

$= 7.5(3-4x)^{-1/2}$

$f'(x) = -3.75(3-4x)^{-3/2}(-4)$

$= 15(3-4x)^{-3/2}$

$f''(x) = -\frac{45}{2}(3-4x)^{-5/2}(-4)$

$\boxed{f''(x) = 90(3-4x)^{-5/2}}$

#20 $y = (4x+1)^6$

$y' = 6(4x+1)^5 \cdot 4$

$= 24(4x+1)^5$

$y'' = 120(4x+1)^4(4)$

$\boxed{y'' = 480(4x+1)^4}$

#22 $y = 3(2x^3+3)^4$

$y' = 12(2x^3+3)^3(6x^2)$

$= (72x^2)(2x^3+3)^3$

$y'' = 144x(2x^3+3)^3 + 3(2x^3+3)^2(6x^2)(72x^2)$

$= 144x(2x^3+3)^3 + 1296x^4(2x^3+3)^2$

$= 144x(2x^3+3)^2[2x^3+3+9x^3]$

$\boxed{y'' = 144x(2x^3+3)^2(11x^3+3)}$

#24 $f(R) = \frac{1-3R}{1+3R}$

$f'(R) = \frac{-3(1+3R) - 3(1-3R)}{(1+3R)^2}$

$= \frac{-3-9R-3+9R}{(1+3R)^2}$

$= \frac{-6}{(1+3R)^2} = -6(1+3R)^{-2}$

$f''(R) = 12(1+3R)^{-3}(3)$

$\boxed{= \frac{36}{(1+3R)^3}}$

#26 $y = \dfrac{x}{\sqrt{1-x^2}}$

$$y' = \frac{\sqrt{1-x^2} - \frac{1}{2}(1-x^2)^{-1/2}(-2x)x}{1-x^2}$$

$$= \frac{\sqrt{1-x^2} + \dfrac{x^2}{\sqrt{1-x^2}}}{1-x^2}$$

$$= \frac{1-x^2+x^2}{\sqrt{1-x^2}} \cdot \frac{1}{1-x^2}$$

$$= \frac{1}{(1-x^2)^{3/2}} = (1-x^2)^{-3/2}$$

$$y'' = -\frac{3}{2}(1-x^2)^{-5/2}(-2x)$$

$$\boxed{y'' = 3x(1-x^2)^{5/2}}$$

#28 $xy + y^2 = 4$

$$y + xy' + 2yy' = 0$$

$$y'(x+2y) = -y$$

$$y' = \frac{-y}{x+2y}$$

$$y'' = \frac{-y'(x+2y) - (1+2y')(-y)}{(x+2y)^2}$$

substitute $y' = \dfrac{-y}{x+2y}$

$$y'' = \frac{\frac{y}{x+2y}(x+2y) + y\left(1+2\left(\frac{-y}{x+2y}\right)\right)}{(x+2y)^2}$$

$$= \left(y + y - \frac{2y^2}{x+2y}\right) \cdot \frac{1}{(x+2y)^2}$$

$$= \left[\frac{2y(x+2y) - 2y^2}{x+2y}\right]\left[\frac{1}{(x+2y)^2}\right]$$

$$= \boxed{\frac{2xy + 2y^2}{(x+2y)^3}}$$

#30

$$4xy = y^2 + 2e^3$$

$$4y + 4xy' = 2yy'$$

$$y'(4x-2y) = -4y$$

$$y' = \frac{-4y}{4x-2y}$$

$$y'' = \frac{-4y'(4x-2y) - (4-2y')(-4y)}{(4x-2y)^2}$$

$$= \left[-4\left(\frac{-4y}{4x-2y}\right)(4x-2y) + 4y\left(4-2\left(\frac{-4y}{4x-2y}\right)\right)\right] \cdot \left[\frac{1}{(4x-2y)^2}\right]$$

$$= \left[16y + 16y + \frac{8y}{4x-2y}\right] \cdot \left[\frac{1}{(4x-2y)^2}\right] = \frac{32y(4x-2y)+8y}{(4x-2y)^3} = \boxed{\frac{128xy - 56y}{(4x-2y)^3}}$$

P. 829

#57 $y = \sin 3x$ show that $\frac{d^2y}{dx^2} = -9y$

$$\frac{dy}{dx} = (\cos 3x)(3)$$
$$= 3\cos 3x$$

$$\frac{d^2y}{dx^2}\ (y'') = -3\sin(3x)(3)$$
$$= -9\sin 3x$$

but $y = \sin 3x$ so

$$\frac{d^2y}{dx^2} = -9\sin 3x = -9y$$ as required.
QED

58 $y = e^{5x}(a+bx)$

$$\frac{dy}{dx} = y' = e^{5x}(5)(a+bx) + be^{5x}$$
$$= 5ae^{5x} + 5bxe^{5x} + be^{5x}$$
$$= e^{5x}(5a + 5bx + b)$$

$$\frac{d^2y}{dx^2} = y'' = e^{5x}(5)(5a+5bx+b) + 5be^{5x}$$
$$= e^{5x}(25a+25bx+5b+5b)$$
$$= e^{5x}(25a+25bx+10b)$$

Now $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y$

$$= e^{5x}(25a+25bx+10b) - 10(e^{5x})(5a+5bx+b) + 25e^{5x}(a+bx)$$
$$= e^{5x}[25a+25bx+10b-50a-50bx-10b+25a+25bx]$$
$$= e^{5x}[50a-50a+50bx-50bx+10b-10b]$$
$$= e^{5x}\cdot 0 = 0$$ As required

QED

P. 697

#18 $y^2 = 4x + 4$

$2yy' = 4$

$y' = \frac{4}{2y} = \frac{2}{y}$

$y^2 = 4 - 4x$

$2yy' = -4$

$y' = \frac{-4}{2y} = \frac{-2}{y}$

y' is the slope of the tangent

WHEN do the curves INTERSECT?

$4x + 4 = 4 - 4x$

$8x = 0$

$x = 0$

$y^2 = 4x + 4$

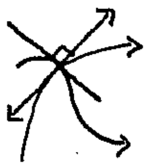
$y^2 = 4$

$y = \pm 2$

AT $(0,-2)$ & $(0,2)$

SLOPE OF TAngents AT $(0,-2)$:

$m_1 = \frac{2}{y} = \frac{2}{-2} = -1$ $\qquad m_2 = -\frac{2}{y} = \frac{-2}{-2} = 1$



$m_1 \cdot m_2 = -1$ curves cross AT right Angles

AT $(0,2)$:

$m_1 = \frac{2}{y} = \frac{2}{2} = 1$ $\qquad m_2 = -\frac{2}{y} = \frac{-2}{2} = -1$

$m_1 \cdot m_2 = -1$ curves cross AT right Angles

#20 p. 697

$y = x^{3/4}$ WHEN $x = 16$

$$y = (\sqrt[4]{16})^3 = (\pm 2)^3 = \pm 8$$

TWO POINTS : $(16, 8)$ & $(16, -8)$

SLOPE OF TANGENT :

$$y' = \frac{3}{4} x^{-1/4} = \frac{3}{4\sqrt[4]{x}}$$

AT the POINT $(16, 8)$: SLOPE is $y' = \frac{3}{4\sqrt[4]{16}} = \frac{3}{8}$

Perpendicular line : $-8/3$

$$y = -8/3\, x + b$$

$$8 = -8/3(16) + b$$

$$\Longrightarrow b = 8 + \frac{8 \cdot 16}{3} = 152/3$$

AT $(16, -8)$ $y' = -3/8$

NORMAL LINE $m = 8/3$ $y = 8/3\, x + b$

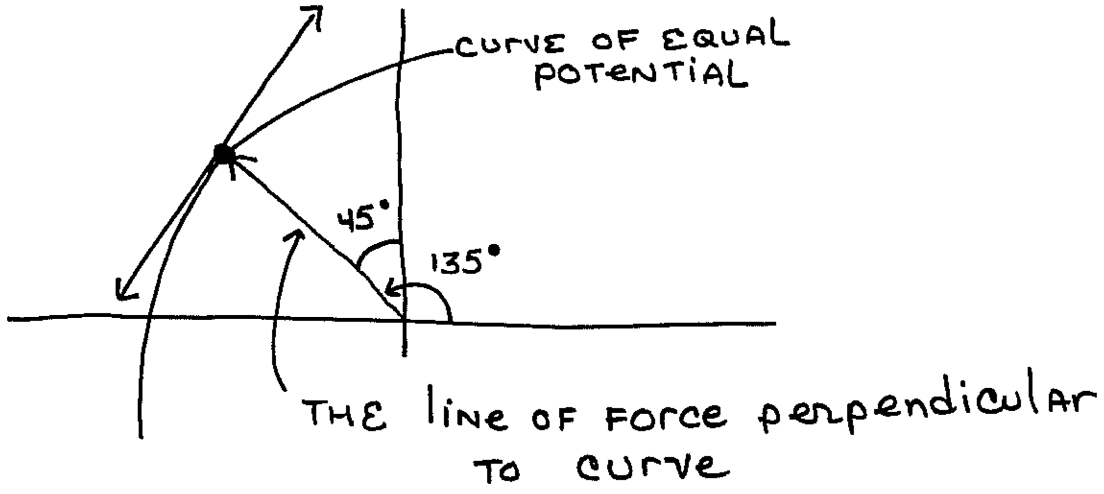
$-8 = 8/3(16) + b$

$$\Longrightarrow b = -8 - \frac{8}{3}(16)$$

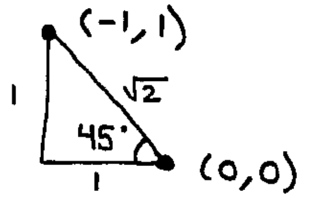
$$= -152/3$$

BONUS #27

$$y = \sqrt{2x^2 + 8}$$



NOTE LINE OF FORCE has A SLope of -1



we will FIND the intersection of the curve & the NORMAL LINE: (Normal & tangent line)

$$y' = \frac{1}{2}(2x^2+8)^{-1/2}(4x) = \frac{2x}{\sqrt{2x^2+8}}$$

WHEN is THIS slope OF tangent 1? (so as to be perpendicular to line with slope -1)

$$\frac{2x}{\sqrt{2x^2+8}} = 1$$

$$2x = \sqrt{2x^2+8}$$

$$4x^2 = 2x^2 + 8$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = \pm 2$$

ONLY $x = 2$ is A sol^N (check)

$$y = \sqrt{2(2)^2 + 8} = \sqrt{16} = 4$$

Point ON TAngent & Normal

$$y = -x + b$$

$$4 = -2 + b$$

$$b = 6$$

$$\boxed{y = -x + 6}$$