Assignment # 7
NYA Electro
SOLUTIONS

TABLE

$f(x)=x^n$	$f'(x)=nx^{n-1}$
$f(x)=K$	$f'(x)=0$
$f(x)=\sin x$	$f'(x)=\cos x$
$f(x)=\cos x$	$f'(x)=-\sin x$
$f(x)=\tan x$	$f'(x)=\sec^2 x$
$f(x)=e^x$	$f'(x)=e^x$
$f(x)=\ln x$	$f'(x)=1/x$
$f(x)=\log_a x$	$f'(x)=\frac{1}{\ln a}\frac{1}{x}$
$f(x)=a^x$	$f'(x)=\ln a\, a^x$
$f(x)=\arctan x$	$f'(x)=\frac{1}{1+x^2}$
$f(x)=\arccos x$	$f'(x)=-\frac{1}{\sqrt{1-x^2}}$
$f(x)=\arcsin x$	$f'(x)=\frac{1}{\sqrt{1-x^2}}$

(1) $f(x)=\sin x \ln 5x$

$$f'(x)=\cos x\ln 5x+\frac{1}{5x}(5)\sin x$$

$$=\boxed{\cos x\ln 5x+\frac{\sin x}{x}}$$

(2) $f(x)=\log_5(xe^x)$

$$f'(x)=\frac{1}{\ln 5}\frac{1}{xe^x}(e^x+xe^x)$$

$$=\frac{1}{\ln 5}\frac{1}{x\cancel{e^x}}\cancel{e^x}(1+x)=\boxed{\frac{1+x}{(\ln 5)x}}$$

(3) $y=\ln x^4\sin^2 x=\ln(x^4)(\sin x)^2$

$$y'=\frac{1}{x^4}4x^3(\sin x)^2+2\sin x\cos x\ln(x^4)$$

$$=\boxed{\frac{4\sin^2 x}{x}+2\sin x\cos x\ln(x^4)}$$

(4) $y=2^{3x^2}$

$$\boxed{y'=(\ln 2)2^{3x^2}(6x)}$$

(5) $y=x^{\cos x}$

$$\ln y=\ln(x^{\cos x})$$

$$\ln y=(\cos x)(\ln x)$$

$$\frac{1}{y}y'=(-\sin x)\ln x+\frac{1}{x}\cos x$$

$$y'=y\left((-\sin x)\ln x+\frac{\cos x}{x}\right)$$

$$y'=\boxed{x^{\cos x}\left(-\sin x(\ln x)+\frac{\cos x}{x}\right)}$$

(6) $y=(\tan x)^{\frac{1}{x}}$

$$\ln y=\frac{1}{x}\ln(\tan x)$$

$$\frac{1}{y}y'=\frac{\frac{1}{\tan x}\sec^2 x(x)-\ln(\tan x)}{x^2}$$

$$y'=\frac{y}{x^2}\left[\frac{x\sec^2 x}{\tan x}-\frac{\tan x(\ln\tan x)}{\tan x}\right]$$

$$y'=\boxed{(\tan x)^{1/x}\left[\frac{x\sec^2 x-\tan x(\ln\tan x)}{x^2\tan x}\right]}$$

(7) $y=\sqrt{x}\,e^{x^2}(x^2+10)^{10}$

Long way:

$$y'=\left(\tfrac{1}{2}x^{-1/2}e^{x^2}+\sqrt{x}e^{x^2}2x\right)(x^2+10)^{10}+10(x^2+10)^9(2x)\sqrt{x}\,e^{x^2}$$

$$=\left(\frac{e^{x^2}}{2\sqrt{x}}+2e^{x^2}x^{3/2}\right)(x^2+10)^{10}+20(x^2+10)^9x^{3/2}e^{x^2}$$

$$=\left(\frac{e^{x^2}+4e^{x^2}x^2}{2\sqrt{x}}\right)(x^2+10)^{10}+\frac{40(x^2+10)^9x^2e^{x^2}}{2\sqrt{x}}$$

$$=\frac{e^{x^2}(x^2+10)^9}{2\sqrt{x}}\left[(1+4x^2)(x^2+10)+40x^2\right]$$

$$=\frac{e^{x^2}(x^2+10)^9}{2\sqrt{x}}\left[x^2+40x^2+4x^4+10+40x^2\right]$$

$$=\boxed{\frac{e^{x^2}(x^2+10)^9(4x^4+81x^2+10)}{2\sqrt{x}}}$$

#7 continued

logarithmic diff.

$y = \sqrt{x}\, e^{x^2} (x^2+10)^{10}$

$\ln y = \ln\sqrt{x} + \ln e^{x^2} + \ln(x^2+10)^{10}$

$\ln y = \frac{1}{2}\ln x + x^2 + 10 \ln(x^2+10)$

$\frac{1}{y} y' = \frac{1}{2}\left(\frac{1}{x}\right) + 2x + \frac{10}{x^2+10}(2x)$

$y' = \boxed{\sqrt{x}\, e^{x^2}(x^2+10)^{10}\left(\frac{1}{2x} + 2x + \frac{20x}{x^2+10}\right)}$

(8) $y = \dfrac{1-xe^x}{x+e^x}$

$y' = \dfrac{[-e^x - xe^x][x+e^x] - [1+e^x][1-xe^x]}{(x+e^x)^2}$

$= \dfrac{[\cancel{-xe^x} - e^{2x} - x^2e^x - \cancel{xe^{2x}}] - [1+e^x - \cancel{xe^{2x}} - \cancel{xe^x}]}{(x+e^x)^2}$

$= \boxed{\dfrac{-e^{2x} - x^2e^x - 1 - e^x}{[x+e^x]^2}}$

(9) $f(x) = 2^{\sin \pi x}$

$f'(x) = \boxed{(\ln 2)\, 2^{\sin\pi x} (\cos\pi x)(\pi)}$

(10) $f(x) = x 3^{-1/x}$

$f'(x) = 3^{-1/x} + x(\ln 3) 3^{-1/x}(x^{-2})$

$= \boxed{3^{-1/x}\left[1 + \frac{\ln 3}{x}\right]}$

(11) $y = \ln \sin x - \frac{1}{2}\sin^2 x$

$y' = \frac{1}{\sin x}\cos x - \frac{1}{2}(2\sin x \cos x)$

$= \boxed{\cot x - \sin x \cos x}$

(12) $y = \sin^{-1}(3x+2)$

$y' = \boxed{\dfrac{1}{\sqrt{1-(3x+2)^2}}(3)}$

(13) $y = \arctan(\sqrt{x^2-1})$

$y' = \dfrac{1}{1+(x^2-1)}\left(\frac{1}{2}(x^2-1)^{-1/2}\right)(2x)$

$= \dfrac{1}{x^2}\left(\dfrac{1}{2\sqrt{x^2-1}}\right)2x = \boxed{\dfrac{1}{x\sqrt{x^2-1}}}$

(14) $f(x) = x \ln(\arctan x)$

$f'(x) = \boxed{\ln(\arctan x) + \dfrac{1}{\arctan x}\left(\dfrac{1}{1+x^2}\right)}$

(15) $f(x) = \cos^{-1}(e^{2x})$

$f'(x) = \dfrac{-1}{\sqrt{1-(e^{2x})^2}}(e^{2x})(2)$

$= \boxed{\dfrac{-2e^{2x}}{\sqrt{1-e^{4x}}}}$

(16) $f(x) = (1+x^2)\arctan(1+x^2)$

$f'(x) = (2x)\arctan(1+x^2) + \dfrac{1}{1+(1+x^2)^2}(2x)(1+x^2)$

$= \boxed{2x\left[\arctan(1+x^2) + \dfrac{(1+x^2)}{2+2x^2+x^4}\right]}$

(17) $f(x) = \dfrac{\arcsin x}{\sqrt{1-x^2}}$

$f'(x) = \dfrac{\frac{1}{\sqrt{1-x^2}}(\sqrt{1-x^2}) - \frac{1}{2}(1-x^2)^{-1/2}(\arcsin x)}{1-x^2}$

$= \left[1 - \dfrac{\arcsin x}{2\sqrt{1-x^2}}\right]\left[\dfrac{1}{1-x^2}\right]$

$= \left[\dfrac{2\sqrt{1-x^2}}{2\sqrt{1-x^2}} - \dfrac{\arcsin x}{2\sqrt{1-x^2}}\right]\left[\dfrac{1}{1-x^2}\right]$

$= \boxed{\dfrac{2\sqrt{1-x^2} - \arcsin x}{2(1-x^2)^{3/2}}}$

(18) $y = x \operatorname{Arccos} x - \sqrt{1-x^2}$

$$y' = \operatorname{Arccos} x + x\left(\frac{-1}{\sqrt{1-x^2}}\right) - \frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$= \boxed{\operatorname{Arccos} x}$$

(19) $y = \sin^2(\cos x)$

$$= [\sin(\cos x)]^2$$

$$y' = \boxed{2[\sin(\cos x)][\cos(\cos x)][-\sin x]}$$

(20) $y = \sqrt[5]{x + \tan x}$

$$y' = \boxed{\frac{1}{5}(x+\tan x)^{-4/5}(\tan x + x\sec^2 x)}$$

(21) $x^2 \cos y + \sin 2y = xy$

$$2x\cos y - (\sin y)y'x^2 + (\cos 2y)(2y') = y + xy'$$

$$y'(-x^2\sin y + 2\cos 2y - x = y - 2x\cos y$$

$$y' = \boxed{\frac{y - 2x\cos y}{-x^2\sin y + 2\cos 2y - x}}$$

(22) $\sin(xy) = x^2 - y$

$$\cos(xy)(y + xy') = 2x - y'$$

$$y\cos xy + x\cos(xy)y' = 2x - y'$$

$$y'(x\cos xy - 1) = 2x - y\cos xy$$

$$y' = \boxed{\frac{2x - y\cos xy}{x\cos xy - 1}}$$

(23) $f(x) = \left(1 - \frac{1}{x^2}\right)^{\sqrt{7}}$

$$f'(x) = \boxed{\sqrt{7}\left(1 - \frac{1}{x^2}\right)^{\sqrt{7}-1}\left(2x^{-3}\right)}$$

(24) $xy^4 + x^2y = x + 3y$

$$y^4 + x4y^3y' + 2xy + x^2y' = 1 + 3y'$$

$$y'(4xy^3 + x^2 - 3) = 1 - y^4 - 2xy$$

$$y' = \boxed{\frac{1 - y^4 - 2xy}{4xy^3 + x^2 - 3}}$$

(25) $f(x) = \sec(1+x^2)$

$$f(x) = [\cos(1+x^2)]^{-1}$$

$$f'(x) = -[\cos(1+x^2)]^{-2}[-\sin(1+x^2)](2x)$$

$$= \frac{\sin(1+x^2)}{(\cos(1+x^2))^2}(2x)$$

$$= \boxed{2x\tan(1+x^2)\sec(1+x^2)}$$

(26) $y = (1 + \cos^2 x)^6$

$$y' = \boxed{6(1+\cos^2 x)^5(2\cos x)(-\sin x)}$$

(27) $y = \sin(\tan(\sqrt{\sin x}))$

$$y' = \boxed{\cos(\tan(\sqrt{\sin x}))(\sec^2(\sqrt{\sin x}))\left(\tfrac{1}{2}(\sin x)^{-1/2}\right)\cos x}$$

(28) $f(x) = \tan^3(3x)$

$$f'(x) = \boxed{3\tan^2(3x)\sec^2(3x)\ (3)}$$

(29) $f(x) = \operatorname{Arctan}(\ln(x^2+2))$

$$f'(x) = \boxed{\frac{1}{1+(\ln(x^2+2))^2}\ \frac{1}{x^2+2}(2x)}$$

(30) $f(x) = (2x-5)^4(8x^2-5)^{-3}$

$$f'(x) = 4(2x-5)^3(2)(8x^2-5)^{-3} + -3(8x^2-5)^{-4}(16x)(2x-5)^4$$

$$= 8(2x-5)^3(8x^2-5)^{-4}\left[(8x^2-5) - 6x(2x-5)\right]$$

$$= \frac{8(2x-5)^3}{(8x^2-5)^4}(8x^2 - 5 - 12x^2 + 30x)$$

$$= \boxed{\frac{8(2x-5)^3(-4x^2+30x-5)}{(8x^2-5)^4}}$$

(31) $f(x) = \sqrt[3]{\sin^2(4x+1)} = [\sin(4x+1)]^{2/3}$

$$f'(x) = \frac{2}{3}[\sin(4x+1)]^{-1/3}[\cos(4x+1)][4]$$

$$= \boxed{\frac{8\cos(4x+1)}{3\sqrt[3]{\sin(4x+1)}}}$$

(32) $g(t)=\left(\frac{t^2}{t+1}\right)^{-6} = \left(\frac{t+1}{t^2}\right)^6$

$$g'(t) = 6\left(\frac{t+1}{t^2}\right)^5\left(\frac{t^2-2t(t+1)}{t^4}\right)$$
$$= 6\left(\frac{t+1}{t^2}\right)^5\left(\frac{-t^2-2t}{t^4}\right)$$
$$= \frac{-6(t+1)^5 t(t+2)}{t^{14}}$$
$$= \boxed{\frac{-6(t+1)^5(t+2)}{t^{13}}}$$

(33) $y = \frac{\sqrt{x}+1}{\sqrt{x}-1}$

$$y' = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - \frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{\sqrt{x}-1}$$
$$= \left(\frac{1}{2}-\frac{1}{2\sqrt{x}}-\frac{1}{2}-\frac{1}{2\sqrt{x}}\right)\frac{1}{\sqrt{x}-1}$$
$$= -\frac{1}{\sqrt{x}}\left(\frac{1}{\sqrt{x}-1}\right)$$
$$= \boxed{\frac{-1}{x-\sqrt{x}}}$$

(34) $h(u) = (U^{-2}+U^{-3})(U^5-2U^2)^4$

$$h'(u) = (-2U^{-3}-3U^{-4})(U^5-2U^2)^4 + 4(U^5-2U^2)^3(5U^4-4U)(U^{-2}+U^{-3})$$
$$= U^{-4}(-2U-3)(U^2)^4(U^3-2)^4 + 4(U^2)^3(U^3-2)^3 U(5U^3-4)U^{-3}(U+1)$$
$$= U^4(-2U-3)(U^3-2)^4 + U^4(4)(U^3-2)^3(5U^3-4)(U+1)$$
$$= U^4(U^3-2)^3\left[(-2U-3)(U^3-2) + 4(5U^3-4)(U+1)\right]$$
$$= U^4(U^3-2)^3\left[-2U^4+4U-3U^3+6+20U^4+20U^3-16U+4\right]$$
$$= \boxed{U^4(U^3-2)^3(18U^4+17U^3-12U+10)}$$

(35) $\tan\left(\frac{x}{y}\right) = x+y$

$$\sec^2(x/y)\left(\frac{y-xy'}{y^2}\right) = 1+y'$$
$$\sec^2(x/y)(y-xy') = y^2+y^2y'$$
$$y\sec^2(x/y) - x\sec^2(x/y)y' = y^2+y^2y'$$
$$y' = \boxed{\frac{y^2-y\sec^2(x/y)}{-x\sec^2(x/y)-y^2}}$$

(36) $\sqrt{x+y} = 1+x^2y^2$

$$\frac{1}{2\sqrt{x+y}}(1+y') = 2xy^2+2yy'x^2$$
$$(1+y') = 4\sqrt{x+y}\,xy^2 + 4\sqrt{x+y}\,x^2yy'$$
$$\boxed{y' = \frac{4\sqrt{x+y}\,xy^2-1}{1-4\sqrt{x+y}\,x^2y}}$$

(37) $y\sin x^2 = x\sin y^2$

$$y'\sin x^2 + \cos x^2(2x)y = \sin y^2 + \cos y^2(2yy')x$$
$$\boxed{y' = \frac{\sin(y^2)-2xy\cos(x^2)}{\sin(x^2)-2xy\cos(y^2)}}$$

(38) $e^{x}y = x+y$

$$e^xy + y'e^x = 1+y' \qquad y' = \boxed{\frac{1-ye^x}{e^x-1}}$$

(39) $xe^y + ye^x = 1$

$$e^y + xe^yy' + y'e^x + ye^x = 0$$
$$y' = \boxed{\frac{-ye^x-e^y}{xe^y+e^x}}$$

(40) $e^{x^2y} = x+y$

$$e^{x^2y}(2xy+x^2y') = 1+y'$$
$$2xye^{x^2y} + x^2e^{x^2y}y' = 1+y'$$
$$y' = \boxed{\frac{1-2xye^{x^2y}}{x^2e^{x^2y}-1}}$$

(41) $y = \frac{x^2+4x+3}{\sqrt{x}} = x^{3/2}+4x^{1/2}+3x^{-1/2}$

$$y' = \boxed{\tfrac{3}{2}x^{1/2} + 2x^{-1/2} - \tfrac{3}{2}x^{-3/2}}$$

(42) $v = t^2 - \frac{1}{\sqrt[4]{t^3}}$

$$= t^2 - t^{-3/4}$$
$$v' = \boxed{2t + \tfrac{3}{4}t^{-7/4}}$$

(43) $R(x) = \sqrt{10}\,x^{-7}$

$$R'(x) = \boxed{-7\sqrt{10}\,x^{-8}}$$

(44) $y = \frac{\ln x}{x^2}$

$y' = \frac{\frac{1}{x}x^2 - 2x\ln x}{x^4}$

$= \frac{1}{x^3} - \frac{2\ln x}{x^3}$

$= \frac{1-2\ln x}{x^3}$

$y'' = \frac{(-2/x)x^3 - 3x^2(1-2\ln x)}{x^6}$

$= \frac{-2}{x^4} - \frac{3(1-2\ln x)}{x^4}$

$= \boxed{\frac{-5+6\ln x}{x^4}}$

(45) $y = x\ln x$

$y' = \ln x + x\left(\frac{1}{x}\right)$

$= \ln x + 1$

$y'' = \boxed{1/x}$

(46) $f(t) = (4t+1)^{1/2}$

$f'(t) = \frac{1}{2}(4t+1)^{-1/2}\,4$

$= 2(4t+1)^{-1/2}$

$f''(t) = -(4t+1)^{-3/2}\cdot 4$

$= \boxed{\frac{-4}{(4t+1)^{3/2}}}$

(47) $x^6 + y^6 = 1$

$6x^5 + 6y^5y' = 0$

$y' = \frac{-6x^5}{6y^5} = \frac{-x^5}{y^5}$

$y'' = \frac{-5x^4y^5 - 5y^4y'(-x^5)}{y^{10}}$

$= \frac{-5x^4y^5 + 5x^5y^4\left(\frac{-x^5}{y^5}\right)}{y^{10}}$

$= \boxed{\frac{-5x^4}{y^5} - \frac{5x^{10}}{y^{11}}}$

(48) $y = (1+2x)^{10}$

$y' = 10(1+2x)^9(2)$

$= 20(1+2x)^9$

$y'' = 180(1+2x)^8(2)$

$= \boxed{360(1+2x)^8}$

(49) $y = \sin x + \sin^2 x$

$y' = \cos x + 2\sin x\cos x$

$= \cos x(1+2\sin x)$

$y'' = -\sin x(1+2\sin x) + 2\cos x(\cos x)$

$= \boxed{-\sin x - 2\sin^2 x + 2\cos^2 x}$

(50) $y = (x^3+1)^{2/3}$

$y' = \frac{2}{3}(x^3+1)^{-1/3}(3x^2)$

$= 2x^2(x^3+1)^{-1/3}$

$y'' = 4x(x^3+1)^{-1/3} - \frac{1}{3}(x^3+1)^{-4/3}(3x^2)(2x^2)$

$= 2x(x^3+1)^{-4/3}\left[2(x^3+1) - x^3\right]$

$= \boxed{2x(x^3+1)^{-4/3}(x^3+2)}$

(51) $H(t) = \tan 3t$

$H'(t) = \sec^2(3t)\cdot 3 = 3(\cos(3t))^{-2}$

$H''(t) = -6\cos(3t)^{-3}(-\sin(3t))(3)$

$= \boxed{\frac{18\sin 3t}{\cos^3(3t)}}$

(52) $\sqrt{x} + \sqrt{y} = 1$

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}y' = 0$

$y' = \frac{-1}{2\sqrt{x}}(2\sqrt{y}) = \frac{-\sqrt{y}}{\sqrt{x}}$

$y'' = \frac{-\frac{1}{2\sqrt{y}}\sqrt{x}\,y' - \frac{1}{2\sqrt{x}}(-\sqrt{y})}{x} = \frac{-\frac{1}{2\sqrt{y}}\sqrt{x}\left(\frac{-\sqrt{y}}{\sqrt{x}}\right) + \frac{\sqrt{y}}{2\sqrt{x}}}{x}$

$= \left(\frac{1}{2} + \frac{\sqrt{y}}{2\sqrt{x}}\right)\frac{1}{x} = \boxed{\frac{1}{2x} + \frac{\sqrt{y}}{2x^{3/2}}}$