ASSIGNMENT #8
NYA ELECTRO
SOLUTIONS

P. 712 #6 $Y = 2 + 6x - 3x^2$

$Y' = 6 - 6x$
$= 6(1 - x)$
$Y' = 0$ WHEN $x = 1$

INTERVAL	$(-\infty, 1)$	$(1, \infty)$
TEST POINT	0	2
Sign of f'	+	−
BEHAVIOR of f	↗	↘

$Y = f(x)$ is INCREASING ON $(-\infty, 1)$
& DECREASING ON $(1, \infty)$

#8 $Y = x^4 - 6x^2$
$= x^2(x^2 - 6)$

$Y' = 4x^3 - 12x$
$= 4x(x^2 - 3)$

$Y' = 0$ AT $x = 0$ & $x = \pm\sqrt{3}$

INTERVAL	$(-\infty, -\sqrt{3})$	$(-\sqrt{3}, 0)$	$(0, \sqrt{3})$	$(\sqrt{3}, \infty)$
TEST pt.	−2	−1	1	2
Sign of f'	−	+	−	+
BEHAVIOR of f	↘	↗	↘	↗

$Y = f(x)$ is INCREASING ON $(-\sqrt{3}, 0)$ & $(\sqrt{3}, \infty)$
& DECREASING ON $(-\infty, -\sqrt{3})$ & $(0, \sqrt{3})$

#10 $Y = 2 + 6x - 3x^2$ HAS A MAX AT $x = 1$

$Y = 2 + 6(1) - 3(1)^2 = 5$

MAX @ $(1, 5)$

#12 $Y = x^4 - 6x^2$ HAS MINIMUMS AT $x = \pm\sqrt{3}$

WHEN $x = \sqrt{3}$ $Y = (\sqrt{3})^4 - 6(\sqrt{3})^2$
$= 9 - 18 = -9$

$x = -\sqrt{3}$ $Y = (-\sqrt{3})^4 - 6(-\sqrt{3})^2$
$= -9$

MINS @ $(-\sqrt{3}, -9)$ & $(\sqrt{3}, -9)$

MAX @ $x = 0$ $Y = (0)^4 - 6(0)^2$
$= 0$

MAX @ $(0, 0)$

#14 $Y' = 6 - 6x$
$Y'' = -6$ SINCE $Y'' < 0$ for all x

THEN Y is CONCAVE DOWN FOR $(-\infty, \infty)$

#16 $Y = X^4 - 6X^2$

$Y' = 4X^3 - 12X$

$Y'' = 12X^2 - 12$

$= 12(X^2 - 1)$

$X = \pm 1$

INTERVALS	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
TEST pt.	-2	0	2
sign of f''	$+$	$-$	$+$
BEHAVIOR of f (CONCAVITY)	$\cup$	$\cap$	$\cup$

$Y = f(x)$ is concave up on $(-\infty, -1)$ & $(1, \infty)$
concave down on $(-1, 1)$

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#22 $Y = 4X^2 - 9$

① INTERCEPTS $X = 0$ $Y = -9$

$Y = 0$ $0 = 4X^2 - 9$

$9 = 4X^2$

$X^2 = 9/4$

$X = \pm\sqrt{9/4} = \pm 3/2$

② Limits at $\pm\infty$

$\lim_{X \to \infty} 4X^2 - 9$ tends to $+\infty$

$\lim_{X \to -\infty} 4X^2 - 9$ tends to $+\infty$

③ VERTICAL ASYMPTOTES

NONE; NO VALUES WHERE Y DOES NOT EXIST

④ domain is $(-\infty, \infty)$

⑤ $Y' = 8X$

$Y' = 0$ WHEN $X = 0$

INTERVAL	$(-\infty, 0)$	$(0, \infty)$
test pt.	-1	1
sign of Y'	$-$	$+$
BEHAVIOR of Y	↘	↗

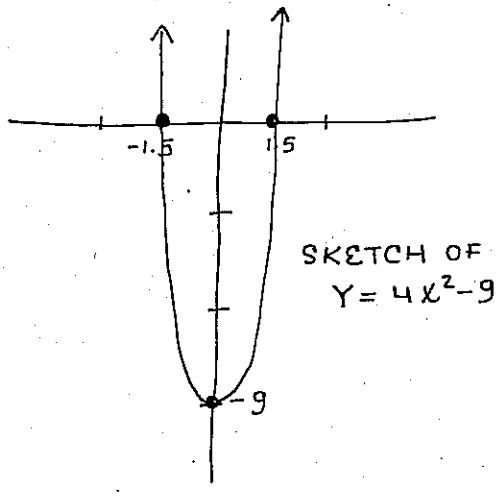
MIN AT $X = 0$

$Y = 4(0)^2 - 9 = -9$

MIN @ $(0, -9)$

⑥ $Y'' = 8$

$Y'' > 0$ so Y is ALWAYS concave up



SKETCH OF $Y = 4X^2 - 9$

#28 $Y = x(x-4)^3$

① x & Y intercepts

$x=0 \quad Y=0$

$Y=0 \quad x=0 \; ; \; x=4$

intercepts $(0,0)$ & $(4,0)$

② limits at $\pm\infty$

$\lim_{x\to\infty} x(x-4)^3 \longrightarrow +\infty$

$\lim_{x\to-\infty} x(x-4)^3 \longrightarrow +\infty$

③ No V.A.s

④ domain $(-\infty, \infty)$

⑤ $Y' = (x-4)^3 + 3(x-4)^2 x$

$= (x-4)^2(x-4+3x)$

$= (x-4)^2(4x-4)$

$= (x-4)^2 4(x-1)$

$Y' = 0 \quad x=4$ OR $x=1$

INTERVAL	$(-\infty,1)$	$(1,4)$	$(4,\infty)$
test pt	0	2	5
sign Y'	−	+	+
BEHAVIOR OF Y	↘	↗	↗

MIN @ $x=1$ $Y = 1(1-4)^3 = -27$

NO MAX

⑥ $Y'' = 2(x-4)4(x-1) + 4(x-4)^2$

$= 4(x-4)[2(x-1)+(x-4)]$

$= 4(x-4)(2x-2+x-4)$

$= 4(x-4)(3x-6)$

$= 12(x-4)(x-2)$

$Y'' = 0 \quad$ AT $x=4$ & $x=2$

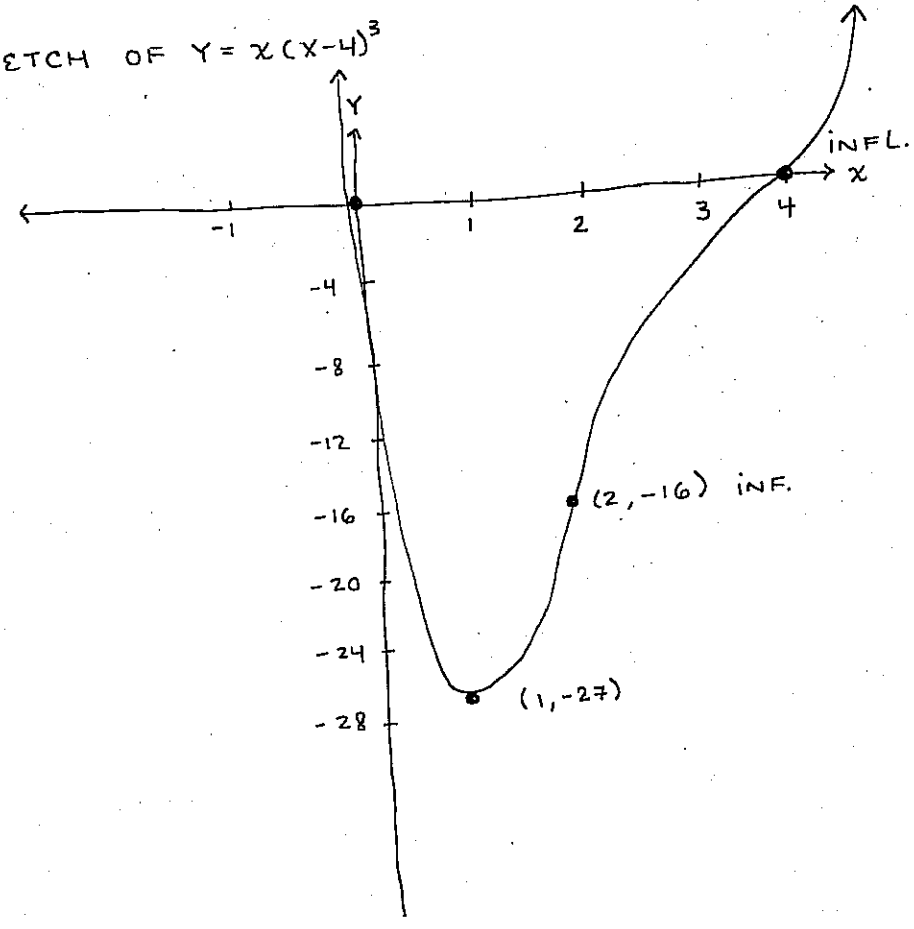
INTERVAL	$(-\infty,2)$	$(2,4)$	$(4,\infty)$
test	0	3	5
sign of Y''	+	−	+
concavity			

INFLECTION POINTS

AT $x=2 \quad Y = 2(2-4)^3 = -16 \quad (2,-16)$

$x=4 \quad Y = 4(4-4)^3 = 0 \quad (4,0)$

SKETCH OF $Y = x(x-4)^3$



#30 $Y = X^5 - 20X^2$

① $Y = X^2(X^3 - 20)$

$X = 0 \quad Y = 0$

$Y = 0 \quad X = 0$ or $X = \sqrt[3]{20} \approx 2.7$ INTERCEPTS $(0,0)$ & $(2.7, 0)$

② $\lim_{X \to \infty} X^2(X^3 - 20) \longrightarrow +\infty$

$\lim_{X \to -\infty} X^2(X^3 - 20) \longrightarrow -\infty$

③ V.A NONE ④ DOMAIN $(-\infty, \infty)$

⑤ $Y' = 5X^4 - 40X$

$= 5X(X^3 - 8)$

$Y' = 0$ WHEN:

$X = 0$ or $X = 2$

INTERVAL	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
TEST PT.	-1	1	3
SIGN OF Y'	$+$	$-$	$+$
BEHAVIOR OF Y	↗	↘	↗

MAX AT $X = 0$
$Y = 0$
$(0, 0)$

MIN AT $X = 2$
$Y = 2^5 - 20(2)^2$
$= 32 - 80 = -48$
$(2, -48)$

⑥ $Y'' = 20X^3 - 40$

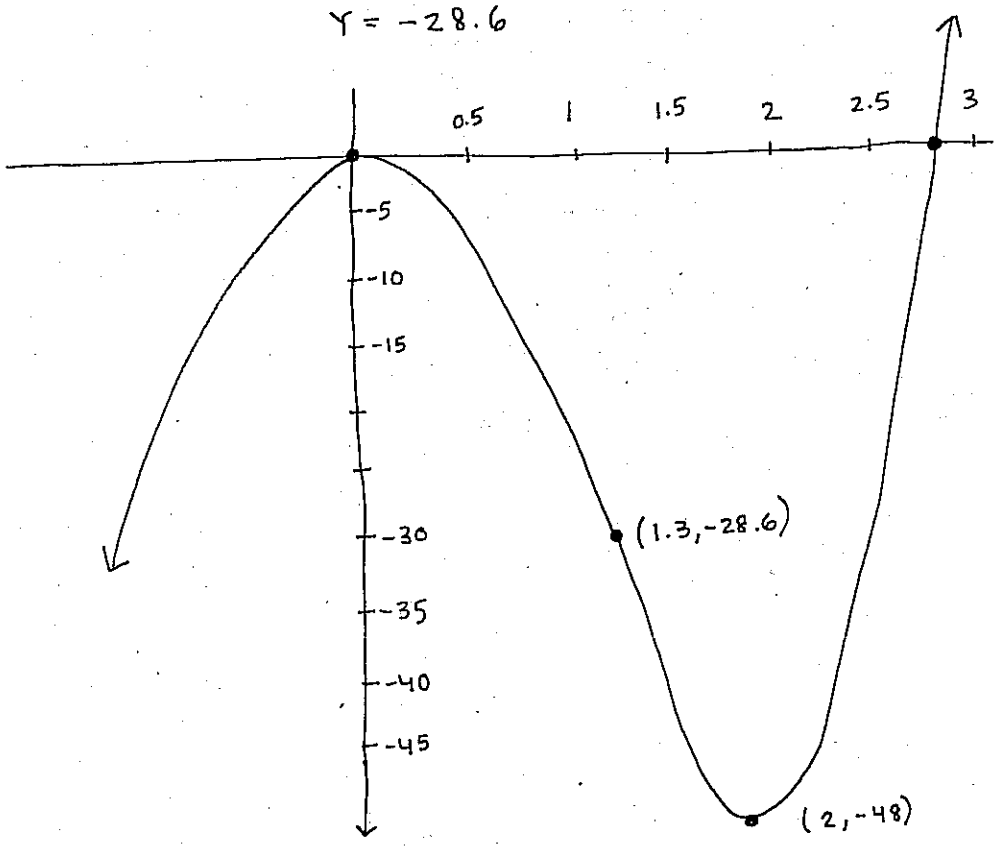
$= 20(X^3 - 2)$

$X = \sqrt[3]{2} \approx 1.3$

INTERVAL	$(-\infty, 1.3)$	$(1.3, \infty)$
test pt	0	2
SIGN OF Y''	$-$	$+$
CONCAVITY	∩	∪

INFL PT AT $X = \sqrt[3]{2} \approx 1.3$

$Y = -28.6$



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3 $Y = \frac{2}{x+1}$

① INTERCEPTS

$x = 0 \quad Y = 2$

$Y = 0$ NEVER

② BEHAVIOR AS $x \to \pm\infty$

$\lim_{x\to\infty} \frac{2/x}{x/x + 1/x} = 0$

$\lim_{x\to-\infty} \frac{2}{x+1} = 0$ (SAME)

③ V.A

$f(x)$ DNE AT $x = -1$

$\lim_{x\to-1} \frac{2}{x+1}$ DNE

FROM LEFT

x	-1.01	-1.001
Y	-20	-200

$\longrightarrow -\infty$

FROM RIGHT

x	-0.9	-0.99
Y	20	200

$\longrightarrow +\infty$

V.A. AT $x = -1$

④ DOMAIN

$(-\infty, \infty)$ MINUS $x = -1$

⑤ $Y' = \frac{-2}{(x+1)^2}$

Y' DNE AT $x = -1$

Y' IS ALWAYS NEGATIVE

SO Y IS ALWAYS DECREASING

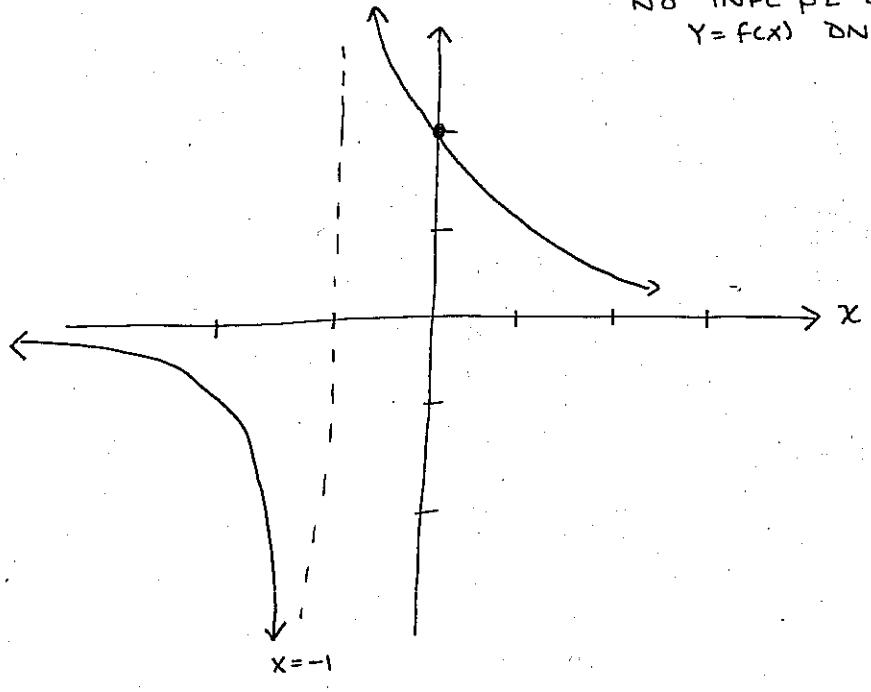
INTERVAL	$(-\infty, -1)$	$(-1, \infty)$
TEST	-2	0
SIGN OF Y'	$-$	$-$
BEHAVIOR OF Y	$\searrow$	$\searrow$

⑥ $Y'' = \frac{4}{(x+1)^3}$

Y'' DNE AT $x = -1$

$(-\infty, -1)$	$(-1, \infty)$	INTERVAL
-2	0	test
$-$	$+$	SIGN OF Y''
$\cap$	$\cup$	CONCAVITY

NO INFL pt BECAUSE $Y = f(x)$ DNE AT $x = -1$



#5 $Y = X^2 + \frac{2}{X}$

① INTERCEPTS

$X = 0$ Y DNE

$Y = 0$ $X^2 = -\frac{2}{X}$

$X^3 = -2$

$X = \sqrt[3]{-2}$

$= -1.26$

② $\lim_{X \to \infty} X^2 + \frac{2}{X} \longrightarrow \infty$

$\lim_{X \to -\infty} X^2 + \frac{2}{X} \longrightarrow \infty$

③ V.A

Y DNE AT $X = 0$

$\lim_{X \to 0} X^2 + \frac{2}{X}$ dNE

FROM LEFX X -0.01 -0.001

$Y \approx -20$ ≈ -200

$\longrightarrow -\infty$

FROM RIGHT X 0.01 0.001

$Y \approx 20$ ≈ 200

$\longrightarrow \infty$

V.A AT $X = 0$

④ DOMAIN $(-\infty, \infty)$ MINUS $X = 0$

⑤ $Y' = 2X - \frac{2}{X^2}$

Y' DNE AT $X = 0$

$Y' = 0$ AT $2X = \frac{2}{X^2}$

$X^3 = 1$ $X = 1$

INTERVALS	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
TEST	-1	$\frac{1}{2}$	2
SIGN OF F'	$-$	$-$	$+$
BEHAVIOR OF $F(X) = Y$	↘	↘	↗

MIN AT $X = 1$

$Y = 3$ $(1, 3)$

⑥ $Y'' = 2 + \frac{4}{X^3}$

Y'' DNE AT $X = 0$

$Y'' = 0$ WHEN $2 = -\frac{4}{X^3}$

$X^3 = -2$

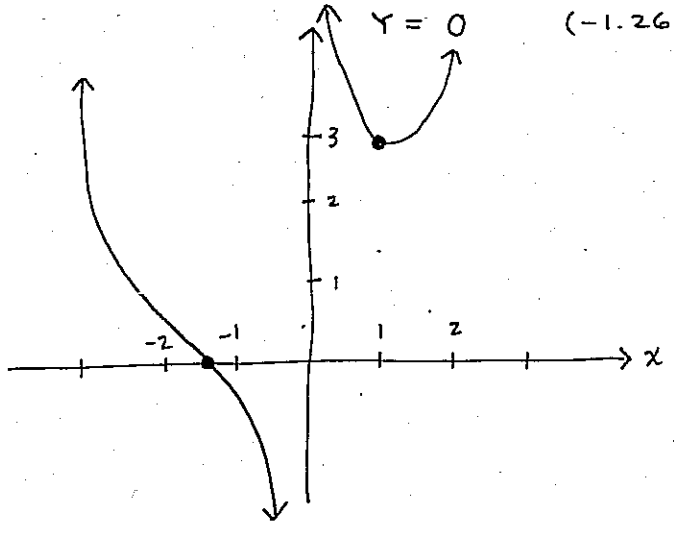
$X = \sqrt[3]{-2}$

≈ -1.26

INTERVALS	$(-\infty, \sqrt[3]{-2})$	$(\sqrt[3]{-2}, 0)$	$(0, \infty)$
TEST	-2	-1	1
SIGN OF Y''	$+$	$-$	$+$
CONCAVITY	∪	∩	∪

INFL. PT AT $X = \sqrt[3]{-2} \approx -1.26$

$Y = 0$ $(-1.26, 0)$



11 $Y = \dfrac{1}{X^2-1}$

① $X=0 \quad Y=-1$

$Y=0 \quad$ never

② $\lim_{X\to\infty} \dfrac{1}{X^2-1} = \lim_{X\to\infty} \dfrac{1/X^2}{X^2/X^2 - 1/X^2} = 0/1 = 0$

$\lim_{X\to-\infty} \dfrac{1}{X^2-1} = 0$

③ V.A

POSSIBLY AT $X=\pm 1$

BECAUSE Y D.N.E AT $X=\pm 1$

$\lim_{X\to 1} \dfrac{1}{X^2-1}$ DNE

$\lim_{X\to -1} \dfrac{1}{X^2-1}$ DNE

V.A.s at $X=\pm 1$

$X=1$ LEFT 0.9 0.99

Y

RIGHT 1.1 1.01

Y

$X=-1$ LEFT -1.01 -1.001

$Y \approx 50$ ≈ 500 $\longrightarrow \infty$

RIGHT -0.9 -0.99

$Y \approx -5$ ≈ -50 $\longrightarrow -\infty$

④ DOMAIN $(-\infty,\infty)$ MINUS ± 1

⑤ $Y' = \dfrac{-2X}{(X^2-1)^2}$ Y' DNE AT $X=\pm 1$

$Y'=0$ AT $X=0$

INTERVAL	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
test	-2	$-\frac{1}{2}$	$\frac{1}{2}$	2
Sign Y'	$+$	$-$	$+$	$-$
BEHAVIOR OF Y	↗	↗	↘	↘

MAX AT $X=0$ $Y=-1$ $(0,-1)$

⑥ $Y'' = \dfrac{-2(X^2-1)^2 - 2(X^2-1)(2X)(-2X)}{(X^2-1)^4}$

$= \dfrac{-2(X^2-1)[X^2-1+2X(-2X)]}{(X^2-1)^4}$

$= \dfrac{-2(-3X^2-1)}{(X^2-1)^3}$

Y'' DNE AT $X=\pm 1$

INTERVAL	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
test	-2	0	2
sign of Y''	$+$	$-$	$+$
concavity	∪	∩	∪

NO INFL. pt.