REVIEW EXERCISES (TEST 1)
SOLUTIONS
(9th Edition OF TEXTBOOK)

(NYA- ELECTROTECH)

P. 656

#5 $f(x) = 3x - 2$

f is CONTINUOUS EVERYWHERE; SINCE IT IS A LINE

#7 $f(x) = \dfrac{2}{x^2 - x} = \dfrac{2}{x(x-1)}$

$f(x)$ does NOT EXIST AT $x = 0$ & $x = 1$; SO IT IS NOT CONTINUOUS THERE

#9 $f(x) = \sqrt{\dfrac{x}{x-2}}$

NOT CONTINUOUS AT $x = 2$ & for $0 < x < 2$ because f does NOT EXIST AT THOSE VALUES (AT $x = 2$ division by 0 & AT $0 < x < 2$ negative under square root)

NOT CONTINUOUS AT $x = 0$ because $\lim\limits_{x \to 0^+} f(x)$ does NOT EXIST

#21 $f(x) = \begin{cases} x^2 & x < 2 \\ 2 & x \geq 2 \end{cases}$

$f(2) = 2$

$\lim\limits_{x \to 2^-} f(x) = 2^2 = 4$

$\lim\limits_{x \to 2^+} f(x) = 2$

$\lim\limits_{x \to 2} f(x)$ does NOT EXIST; f is NOT CONTINUOUS AT $x = 2$

#23 $f(x) = \begin{cases} \dfrac{2x^2 - 18}{x - 3} & x < 3 \text{ or } x > 3 \\ 12 & x = 3 \end{cases}$

① $f(3) = 12$

② $\lim\limits_{x \to 3} f(x) = \lim\limits_{x \to 3} \dfrac{2x^2 - 18}{x - 3} = \lim\limits_{x \to 3} \dfrac{2(x^2 - 9)}{(x-3)} = \lim\limits_{x \to 3} \dfrac{2(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} = 12$

③ $f(3) = \lim\limits_{x \to 3} f(x)$ THEREFORE f is CONTINUOUS AT $x = 3$

#33 $\lim\limits_{x\to 0} \frac{x^2+x}{x} = \lim\limits_{x\to 0} \frac{\cancel{x}(x+1)}{\cancel{x}} = 1$

#35 $\lim\limits_{x\to -1} \frac{x^2-1}{3x+3} = \lim\limits_{x\to -1} \frac{\cancel{(x+1)}(x-1)}{3\cancel{(x+1)}} = \frac{-1-1}{3} = -2/3$

#39 $\lim\limits_{x\to 1} \frac{(2(x)-1)^2-1}{2x-2} = \lim\limits_{x\to 1} \frac{4x^2-4x}{2x-2} = \lim\limits_{x\to 1} \frac{4x\cancel{(x-1)}}{2\cancel{(x-1)}} = 2$

#43 $\lim\limits_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \lim\limits_{x\to 1} \frac{\sqrt{x}-1}{x-1}\,\frac{(\sqrt{x}+1)}{(\sqrt{x}+1)} = \lim\limits_{x\to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x}+1)} = \frac{1}{2}$

#45 $\lim\limits_{x\to \infty} \frac{3x^2+4.5}{x^2-1.5} = \lim\limits_{x\to \infty} \frac{3x^2/x^2 + 4.5/x^2}{x^2/x^2 - 1.5/x^2} = \lim\limits_{x\to \infty} \frac{3+\cancel{4.5/x^2}\to 0}{1-\cancel{1.5/x^2}\to 0} = 3$

#46 $\lim\limits_{x\to \infty} \frac{x-1}{7x+4} = \lim\limits_{x\to \infty} \frac{x/x - 1/x}{7x/x + 4/x} = \lim\limits_{x\to \infty} \frac{1-\cancel{1/x}\to 0}{7+\cancel{4/x}\to 0} = 1/7$

P. 664

#13 $f(x) = 8x - 2x^2$

$$f'(x) = \lim_{h\to 0} \frac{[8(x+h) - 2(x+h)^2] - [8x - 2x^2]}{h}$$

$$= \lim_{h\to 0} \frac{8x + 8h - 2(x^2+2xh+h^2) - 8x + 2x^2}{h}$$

$$= \lim_{h\to 0} \frac{8h - 4xh - 2h^2}{h}$$

$$= \lim_{h\to 0} 8 - 4x - 2h = 8 - 4x$$

#19 $f(x) = x + \frac{4}{3x}$

$$f'(x) = \lim_{h\to 0} \frac{\left[x+h+\frac{4}{3(x+h)}\right] - \left[x + \frac{4}{3x}\right]}{h}$$

$$= \lim_{h\to 0} \left[h + \frac{4}{3x+3h} - \frac{4}{3x}\right]\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \left[h + \frac{\cancel{12x} - \cancel{12x} - 12h}{(3x+3h)3x}\right]\frac{1}{h}$$

$$= \lim_{h\to 0} 1 - \frac{12}{(3x+3h)(3x)} = 1 - \frac{12}{9x^2} = 1 - \frac{4}{3x^2}$$

#39 $y=\sqrt{x+1}$

$$y' = \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \left(\frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}\right)$$

$$= \lim_{h\to 0} \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \frac{1}{\sqrt{x+1} + \sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}$$

P. 668

#15 $s = 48t + 12$

$s' = v = 48$

#17 $s = 12t^2 - t^3$

$s' = v = 24t - 3t^2$

P. 691

#21 $y = 2x^7 - 3x^2 + 5$

$y' = 14x^6 - 6x$

#23 $y = 4\sqrt{x} - \frac{3}{x} + \sqrt{3}$

$= 4x^{1/2} - 3x^{-1} + \sqrt{3}$

$y' = 2x^{-1/2} + 3x^{-2} = \frac{2}{\sqrt{x}} + \frac{3}{x^2}$

#25 $f(y) = \frac{3y}{1-5y}$

$$f'(y) = \frac{3(1-5y) - (-5)(3y)}{(1-5y)^2}$$

$$= \frac{3 - 15y + 15y}{(1-5y)^2}$$

$$= \frac{3}{(1-5y)^2}$$

QUOTIENT RULE

#27 $y = (2-7x)^4$

$y' = 4(2-7x)^3(-7)$

$= -28(2-7x)^3$

CHAIN RULE

#29 $y = \frac{3\pi}{(5-2x^2)^{3/4}}$

$y = 3\pi(5-2x^2)^{-3/4}$

$y' = -\frac{9\pi}{4}(5-2x^2)^{-7/4} \cdot (-4x)$

$= 9\pi(5-2x^2)^{-7/4}$

CHAIN RULE

31 $\quad v = \sqrt{1 + \sqrt{1 + \sqrt{1 + 8s}}}$

$$= \left(1 + \left(1 + (1+8s)^{1/2}\right)^{1/2}\right)^{1/2}$$

$$v' = \frac{1}{2}\left(1+\left(1+(1+8s)^{1/2}\right)^{1/2}\right)^{-1/2} \cdot \left(\frac{1}{2}\left(1+(1+8s)\right)^{-1/2}\right) \cdot \frac{1}{2}(1+8s)^{-1/2} \cdot 8$$

CHAIN RULE 3 times

33 $\quad y = \dfrac{\sqrt{4x+3}}{2x} = \dfrac{(4x+3)^{1/2}}{2x}$

$$y' = \frac{\frac{1}{2}(4x+3)^{-1/2} \cdot 4 \cdot 2x - 2(4x+3)^{1/2}}{4x^2}$$

$$= \frac{\dfrac{4x}{\sqrt{4x+3}} - 2\sqrt{4x+3}}{4x^2} = \frac{4x - 2(4x+3)}{\sqrt{4x+3}} \cdot \frac{1}{4x^2}$$

$$= \frac{-4x-6}{\sqrt{4x+3}\,4x^2} = \frac{-2(2x+3)}{4x^2\sqrt{4x+3}} = \frac{-(2x+3)}{2x^2\sqrt{4x+3}}$$

CHAIN RULE & QUOTIENT RULE

48 $\quad y = \dfrac{x}{x^2+1}$

$$m = \text{slope of tangent line} = y' = \frac{(x^2+1) - 2x(x)}{x^2+1} = \frac{-x^2+1}{x^2+1} = \frac{-(x^2-1)}{x^2+1}$$

$m = 0$ WHEN $\dfrac{-(x^2-1)}{x^2+1} = 0$

$$-(x+1)(x-1) = 0$$

tangent line is HORIZONTAL At $x = \pm 1$

AT $x = 1 \quad y = \frac{1}{2}$ $\qquad$ AT $x = -1 \quad y = -\frac{1}{2}$

THE points Are $(1, \frac{1}{2})$ & $(-1, -\frac{1}{2})$

P. 828

3 $U = 0.2 \tan\sqrt{3-2v}$

$= 0.2 \tan (3-2v)^{1/2}$

$U' = 0.2 \left(\sec^2(3-2v)^{1/2}\right) \cdot \frac{1}{2}(3-2v)^{-1/2} \cdot -2$ (2 CHAIN RULES)

$= \dfrac{-0.2 \sec^2(3-2v)^{1/2}}{\sqrt{3-2v}}$

7 $y = 3\cos^4 x^2$

$= 3(\cos x^2)^4$

$y' = 12(\cos x^2)^3 (-\sin x^2)(2x)$ (2 CHAIN RULES)

$= -24x(\cos^3 x^2)(\sin x^2)$

9 $y = \left(e^{-x-3}\right)^2$

$y = e^{-2x-6}$

$y' = e^{-2x-6} \cdot (-2)$ (CHAIN RULE)

10 $y = 0.5e^{\sin 2x}$

$y' = 0.5e^{\sin 2x}(\cos 2x) \cdot 2$

$= e^{\sin 2x}(\cos 2x)$ (CHAIN RULE X2)

11 $y = 3\ln(x^2+1)$

$y' = \dfrac{3}{x^2+1}(2x) = \dfrac{6x}{x^2+1}$ (CHAIN rule)

19 $y = \dfrac{\cos^2 x}{e^{3x}+\pi^2} = \dfrac{(\cos x)^2}{e^{3x}+\pi^2}$

$y' = \dfrac{2\cos x(-\sin x)(e^{3x}+\pi^2) - e^{3x}(3)(\cos x)^2}{(e^{3x}+\pi^2)^2}$ (quotient rule CHAIN rule X2)

$= \dfrac{-2\sin x\cos x(e^{3x}+\pi^2) - 3e^{3x}\cos^2 x}{(e^{3x}+\pi^2)^2}$

#27 $y = \ln^2(3+\sin x)$

$y = (\ln(3+\sin x))^2$

$y' = 2\ln(3+\sin x)\left(\frac{1}{3+\sin x}\right)(\cos x)$ (CHAIN RULE X 2)

#30 $y = 5e^{3x}\ln x$

$y' = 5e^{3x}\cdot 3\ln x + \frac{1}{x}(5e^{3x})$ (product rule CHAIN rule)

$= 15e^{3x}\ln x + \frac{5e^{3x}}{x}$

$= e^{3x}\left(15\ln x + \frac{5}{x}\right)$

#31 $y = \sqrt{\sin 2x + e^{4x}}$

$= (\sin 2x + e^{4x})^{1/2}$

$y' = \frac{1}{2}(\sin 2x + e^{4x})^{-1/2}\cdot((\cos 2x)(2) + e^{4x}(4))$

$= \frac{\cancel{2}[\cos 2x + 2e^{4x}]}{\cancel{2}\sqrt{\sin 2x + e^{4x}}} = \frac{\cos 2x + 2e^{4x}}{\sqrt{\sin 2x + e^{4x}}}$

#35 $r = 0.5t(e^{2t}+1)(e^{-2t}-1)$

$= 0.5t(e^{2t}e^{-2t} + e^{-2t} - e^{2t} - 1)$ SIMPLIFY FUNCTION FIRST

$= 0.5t(1 + e^{-2t} - e^{2t} - 1)$

$= 0.5t(e^{-2t} - e^{2t})$

$r' = 0.5(e^{-2t} - e^{2t}) + 0.5t(e^{-2t}(-2) - e^{2t}(2))$

$= 0.5e^{-2t} - 0.5e^{2t} - te^{-2t} - te^{2t}$

$= e^{-2t}(0.5 - t) + e^{2t}(-0.5 - t)$

#45 $y = 4\cos^2(x^2)$

$\text{slope} = m = 8\cos(x^2)(-\sin x^2)(2x)$

AT $x = 1 \implies m = 8\cos(1)(-\sin(1))(2)$

$= -16\cos 1 \sin 1$

$= -7.27$

POINT : WHEN $x=1$ $\quad y = 4[\cos(1)]^2 = 1.17$

$y = mx + b$

$y = -7.27x + b$

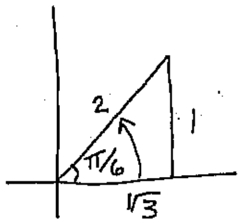
$1.17 = -7.27(1) + b$

$b = 8.44$ $\qquad y = -7.27x + 8.44$

#46 $y = \ln\cos x \qquad x = \pi/6$

$m = y' = \frac{1}{\cos x}(-\sin x)$

AT $x = \pi/6$ $\quad y' = \frac{-\sin(\pi/6)}{\cos(\pi/6)} = \frac{-1/2}{\sqrt{3}/2} = -1/\sqrt{3}$



POINT : WHEN $x = \pi/6$

$y = \ln(\sqrt{3}/2) = -0.144$

$y = mx + b$

$-0.144 = (-1/\sqrt{3})(\pi/6) + b$

$b = 0.16$

$y = -\frac{1}{\sqrt{3}}x + 0.16$