REVIEW FOR TEST 2
NYA ELECTROTECH
SOLUTIONS

p. 828 (9th edition)

(1) $y = 3\cos(4x-1)$

$y' = -3(\sin(4x-1))(4)$

(3) $u = 0.2\tan\sqrt{3-2v}$

$u' = 0.2(\sec^2\sqrt{3-2v})\left(\frac{1}{2}(3-2v)^{-1/2}\right)(-2)$

(5) $y = \csc^2(3x+2)$

$= \dfrac{1}{\sin^2(3x+2)} = (\sin(3x+2))^{-2}$

$y' = -2(\sin(3x+2))^{-3}(\cos(3x+2))(3)$

(7) $y = 3\cos^4(x^2)$

$= 3(\cos x^2)^4$

$y' = 12\cos^3 x^2(-\sin x^2)(2x)$

(9) $y = (e^{x-3})^2 = e^{2x-6}$

$y' = e^{2x-6}(2)$

(11) $y = 3\ln(x^2+1)$

$y' = \dfrac{3}{x^2+1}(2x)$

(13) $y = 10\tan^{-1}(x/5)$

$y' = \dfrac{10}{1+(x/5)^2}\left(\frac{1}{5}\right)$

$= \dfrac{2}{1+(x/5)^2} = \dfrac{2}{1+x^2/25} = \dfrac{50}{25+x^2}$

(15) $\theta = \ln\sin^{-1}(0.1t)$

$\theta' = \dfrac{1}{\sin^{-1}0.1t}\left(\dfrac{1}{\sqrt{1-(0.1t)^2}}\right)(0.1)$

(17) $y = \sqrt{\csc 4x + \cot 4x}$

$= \left(\dfrac{1}{\sin 4x} + \dfrac{1}{\tan 4x}\right)^{1/2}$

$y' = \frac{1}{2}\left((\sin 4x)^{-1} + (\tan 4x)^{-1}\right)^{-1/2} \cdot$

$\left(-(\sin 4x)^{-2}(\cos 4x)4 - (\tan 4x)^{-2}(\sec^2 4x)4\right)$

(19) $y = 7\ln(x-e^{-x})^2$

$= 14\ln(x-e^{-x})$

$y' = \dfrac{14}{x-e^{-x}}(1+e^{-x})$

(21) $y = \dfrac{\cos^2 x}{e^{3x}+\pi^2}$

$y' = \dfrac{2\cos x(-\sin x)(e^{3x}+\pi^2) - 3e^{3x}\cos^2 x}{(e^{3x}+\pi^2)^2}$

(23) $v = \dfrac{u^2}{\tan^{-1}2u}$

$v' = \dfrac{2u\tan^{-1}2u - \frac{1}{1+(2u)^2}(2)u^2}{(\tan^{-1}2u)^2}$

$= \dfrac{(1+4u^2)2u\tan^{-1}2u - 2u^2}{(1+4u^2)(\tan^{-1}2u)^2}$

(25) $y = \ln(\csc x^2)$

$= \ln\left(\dfrac{1}{\sin x^2}\right)$

$= \ln 1 - \ln\sin x^2$

$= 0 - \dfrac{1}{\sin x^2}(\cos x^2)2x$

$= -2x\cot x^2$

(27) $y = \ln^2(3+\sin x)$

$y' = 2\ln(3+\sin x)\left(\frac{1}{3+\sin x}\right)(\cos x)$

(29) $L = 0.1e^{-2t}\sec\pi t$

$= \dfrac{0.1}{e^{2t}\cos\pi t} = 0.1\left(e^{2t}\cos\pi t\right)^{-1}$

$L' = -0.1(e^{2t}\cos\pi t)^{-2}(2e^{2t}\cos\pi t + \pi\sin\pi t\, e^{2t})$

(31) $y = \sqrt{\sin 2x + e^{4x}}$

$y' = \frac{1}{2}(\sin 2x + e^{4x})^{-1/2}(2\cos 2x + 4e^{4x})$

(33) $\tan^{-1}(y/x) = x^2e^y$

$\dfrac{1}{1+(y/x)^2}\left(\dfrac{y'x - y}{x^2}\right) = 2xe^y + e^y y' x^2$

$\dfrac{x^2}{x^2+y^2}\left(\dfrac{y'x-y}{x^2}\right) = 2xe^y + x^2e^y y'$

$y'x - y = (x^2+y^2)(2xe^y + x^2e^y y')$

$y'x - y = 2x^3e^y + x^4e^y y' + 2xy^2e^y + x^2y^2e^y y'$

$y'(x - x^4e^y - x^2y^2e^y) = y + 2x^3e^y + 2xy^2e^y$

$y' = \dfrac{y + 2x^3e^y + 2xy^2e^y}{x - x^4e^y - x^2y^2e^y}$

(35) $r = 0.5t(e^{2t}+1)(e^{-2t}-1)$

$= 0.5t(e^0 + e^{-2t} - e^{2t} - 1)$

$= 0.5t(e^{-2t} - e^{2t})$

$r' = 0.5(e^{-2t} - e^{2t}) + (-2e^{-2t} - 2e^{2t})0.5t$

(37) $\ln xY = 1 - Ye^{-x}$

$\frac{1}{xY}(Y + xY') = -Y'e^{-x} + e^{-x}Y$

$Y + xY' = -Y'xYe^{-x} + xY^2e^{-x}$

$Y'(x + xYe^{-x}) = xY^2e^{-x} - Y$

$Y' = \dfrac{xY^2e^{-x} - Y}{x + xYe^{-x}}$

(39) $Y = x\cos^{-1}x - \sqrt{1-x^2}$

$Y' = \cos^{-1}x - \dfrac{x}{\sqrt{1-x^2}} - \frac{1}{2}(1-x^2)^{-1/2}(-2x)$

$= \cos^{-1}x - \dfrac{x}{\sqrt{1-x^2}} + \dfrac{x}{\sqrt{1-x^2}}$

$= \cos^{-1}x$

P. 683

(15) $Y = 2.25(7-4x^3)^8$

$Y' = 18(7-4x^3)^7(-12x^2)$

P. 686

\# (17) $Y + 3xY - 4 = 0$

$Y' + 3Y + 3xY' = 0$

$Y'(1+3x) = -3Y$

$Y' = \dfrac{-3Y}{1+3x}$

(19) $x^2 = \dfrac{x-Y}{x+Y}$

$x^2(x+Y) = x - Y$

$x^3 + x^2Y = x - Y$

$3x^2 + 2xY + x^2Y' = 1 - Y'$

$Y'(x^2+1) = 1 - 3x^2 - 2xY$

$Y' = \dfrac{1 - 3x^2 - 2xY}{x^2+1}$

(23) $(2Y-X)^4 = y+3-x^2$

$4(2Y-X)^3(2Y'-1) = y'-2x$

$8(2Y-X)^3Y' - 4(2Y-X)^3 = Y'-2X$

$Y'\left(8(2Y-X)^3-1\right) = -2X+4(2Y-X)^3$

$$Y' = \frac{-2X+4(2Y-X)^3}{8(2Y-X)^3-1}$$

(75) $$f = \frac{1}{2\pi\sqrt{C(L+2)}}$$

$$\frac{df}{dL} = \frac{1}{2\pi}\left(-\tfrac{1}{2}(C(L+2))^{-3/2}\right)(C)$$

$$= -\frac{C}{4\pi\left(\sqrt{C(L+2)}\right)^3}$$

p. 692

\#(53) $Y = 7x^4 - x^3$

$Y' = 28x^3 - 3x^2$

m AT $(-1,8) = 28(-1)^3 - 3(-1)^2$

$= -28-3$

$= -31$

(55) $$Y = \frac{1}{\sqrt{3x^2+3}} = (3x^2+3)^{-1/2}$$

$Y' = -\frac{1}{2}(3x^2+3)^{-3/2}(6x)$

$$= \frac{-6x}{2(3x^2+3)^{3/2}}$$

parallel to x-axis $Y'=0$

when $x=0$ $Y = \frac{1}{\sqrt{3}}$

$(0, 1/\sqrt{3})$

(62) $s = 8t - t^2$

$v = s' = 8-2t$

when is $v=4$

$4 = 8-2t$

$-4 = -2t$

$t = 2$

p. 731

\#(1) $y = 3x - x^2$

$y' = 3-2x$ at $(-1,-4)$

$y' = 3-2(-1)$

$= 5$

$y = 5x+b$

$-4 = 5(-1)+b$

$b = 1$ $y = 5x+1$

(3) $x^2 - 4y^2 = 9$ AT $(5,2)$

$2x - 8yy' = 0$

$y' = \frac{2x}{8Y}$ $y' = \frac{2(5)}{8(2)} = \frac{10}{16} = 5/8$

$y = -8/5x + b$

$2 = -8/5(5) + b$

$b = 10$ $Y = -8/5x + 10$

(5) $Y = \sqrt{x^2+3}$ slope $1/2$

$Y' = \frac{1}{2}(x^2+3)^{-1/2}2x$

$\frac{1}{2} = \frac{1}{2}(x^2+3)^{-1/2}2x$

$\sqrt{x^2+3} = 2x$

$x^2+3 = 4x^2$

$3 = 3x^2$

$x^2 = 1$

$x = \pm 1$ $Y = 2$

$Y = 1/2x + b$

$2 = 1/2(1) + b$

$b = 3/2$ $Y = 1/2x + 3/2$

17 $y = 4x^2 + 16x$

① Intercepts

$x=0 \quad y=0$

$y = 4x(x+4)$

$y=0 \quad x=0 \quad x=-4$

② $\lim_{x\to\infty} 4x^2+16x \to \infty$

$\lim_{x\to-\infty} 4x^2+16x \to \infty$

③ No V.A.s

④ Domain: $(-\infty,\infty)$

⑤ $y' = 8x+16$
$= 8(x+2)$

$y'=0 \quad x=-2$

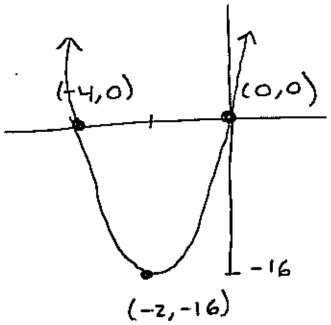
Interval	$(-\infty,-2)$	$(-2,\infty)$
Test	-3	0
sign of y'	$-$	$+$
Behavior of y	↘	↗

Min at $x=-2$ $y=-16$
$(-2,-16)$

⑥ $y'' = 8$

$y'' > 0$

so y is always concave up



(19) $y = 27x - x^3$

① Intercepts

$x=0 \quad y=0$

$y = x(27-x^2)$

$y=0 \quad x=0, \pm\sqrt{27} \approx 5.2$

② $\lim_{x\to\infty} x(27-x^2) \to -\infty$

$\lim_{x\to-\infty} x(27-x^2) \to +\infty$

③ No V.As

④ Domain $(-\infty,\infty)$

⑤ $y' = 27-3x^2$
$= 3(9-x^2)$

$y'=0$ when $x=\pm 3$

Interval	$(-\infty,-3)$	$(-3,3)$	$(3,\infty)$
Test	-4	0	4
sign y'	$-$	$+$	$-$
Behavior of y	↘	↗	↘

Min. at $x=-3$
$y=-108$

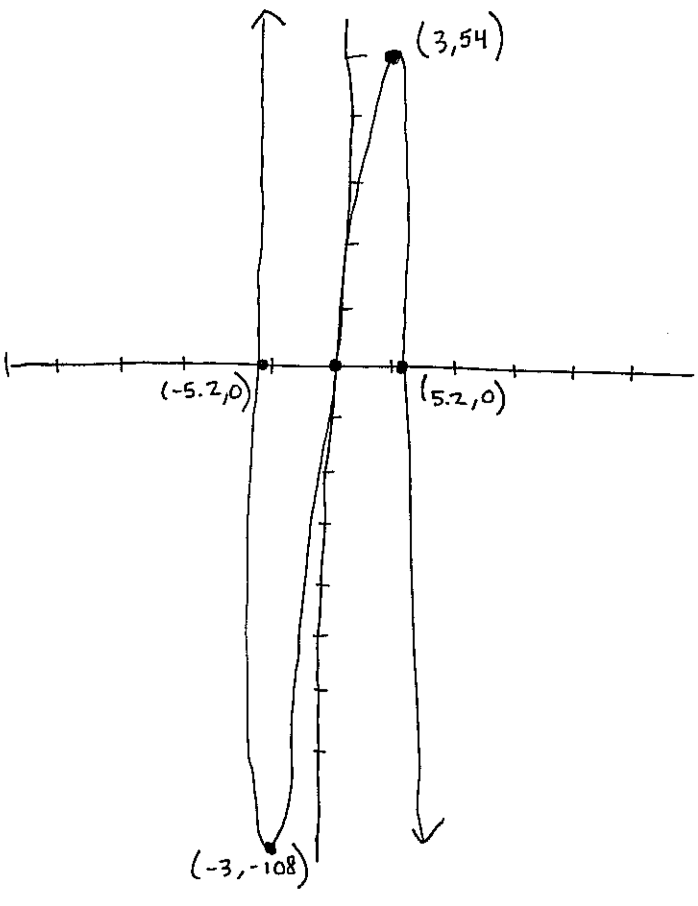
Max $x=3$
$y=54$

⑥ $y'' = -6x$

$y''=0$ when $x=0$

Interval	$(-\infty,0)$	$(0,\infty)$
sign y''	-1	1
Test	$+$	$-$
Concavity	∪	∩

Infl. pt at $x=0$
$y=0$



20 $\quad y = x(6-x)^3$

① INTERCEPTS

$x=0 \quad x=6$ WHEN $Y=0$

$x=0 \quad y=0$

$(0,6)\ (0,0)$

② $\lim\limits_{x\to\infty} x(6-x)^3 \to -\infty$

$\lim\limits_{x\to-\infty} x(6-x)^3 \to -\infty$

③ NO V.A.S

④ DOMAIN $(-\infty,\infty)$

⑤ $Y' = (6-x)^3 + 3(6-x)^2(-x)$

$= (6-x)^2[6-x-3x]$

$= (6-x)^2(-4x+6)$

$= (6-x)^2\, 2(-2x+3)$

$Y'=0 \quad x=6 \quad x=3/2$

INTERVAL	$(-\infty,3/2)$	$(3/2,6)$	$(6,\infty)$
Test pt	0	2	7
Sign OF Y'	+	—	—
BEHAVIOR OF Y	↗	↘	↘

MAX AT $x=3/2$

$Y = 3/2\,(9/2)^3 = 136.7$

⑥ $Y'' = 2(6-x)(-1)(-4x+6) - 4(6-x)^2$

$= (6-x)(-2(-4x+6) - 4(6-x))$

$= (6-x)(8x-12-24+4x)$

$= (6-x)(12x-36)$

$= (6-x)(12)(x-3)$

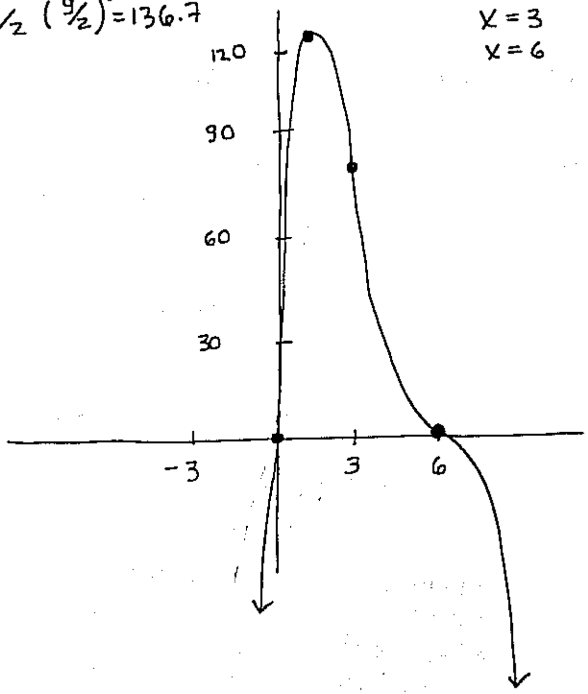
$x=6$ OR $x=3$ WHEN $Y''=0$

INTERVALS	$(-\infty,3)$	$(3,6)$	$(6,\infty)$
Test	0	4	7
Sign OF Y''	—	+	—
BEHAVIOR OF Y	$\cap$	$\cup$	$\cap$

INFLECTION pt

$x=3 \quad Y=81$

$x=6 \quad Y=0$



(21) $Y = x^4 - 32x$

① INTERCEpts

$x=0 \quad Y=0$

$Y=0 \quad x(x^3-32)=0$

$x=0 \quad x=\sqrt[3]{32} \approx 3.2$

② $\lim\limits_{x\to\infty} x^4-32x \to \infty$

$\lim\limits_{x\to-\infty} x^4-32x \to \infty$

③ NO V.A.S

④ DOMAIN $(-\infty,\infty)$

⑤ $Y' = 4X^3 - 32$
$= 4(X^3 - 8)$

$Y' = 0 \quad X = 2$

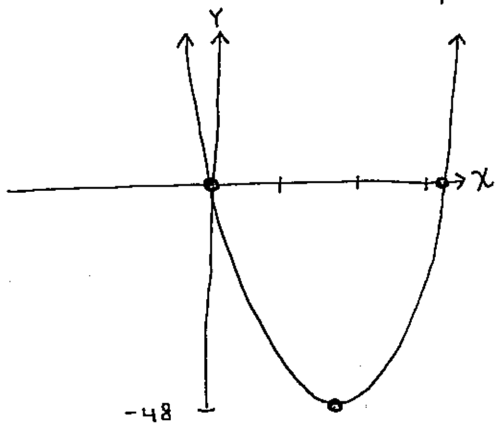
INTERVAL	$(-\infty, 2)$	$(2, \infty)$
SIGN OF Y'	$-$	$+$
TEST	1	3
BEHAVIOR OF Y	↘	↗

MIN AT $X = 2$
$Y = -48$

⑥ $Y'' = 4(3X^2)$
$Y'' = 0$ WHEN $X = 0$

INTERVAL	$(-\infty, 0)$	$(0, \infty)$
SIGN OF Y''	$+$	$+$
TEST PT	-1	1
CONCAVITY	∪	∪

NO INFL. PTS.



㉔ $Y = X^3 + 3/X$

① INTERCEPTS
$X = 0$ IMPOSSIBLE
$0 = X^3 + 3/X$
$0 = X^4 + 3$
$Y = 0 \quad X = \pm\sqrt[4]{3}$
$\approx \pm 1.3$

② $\lim_{X \to \infty} X^3 + 3/X \to \infty$
$\lim_{X \to -\infty} X^3 + 3/X \to -\infty$

③ Y DNE AT $X = 0$
$\lim_{X \to 0^-} Y \to -\infty$
$\lim_{X \to 0^+} Y \to +\infty$

V.A. AT $X = 0$

④ DOMAIN
$\mathbb{R} \setminus \{0\}$

⑤ $Y' = 3X^2 - 3/X^2$

$Y' = 0$

$0 = 3X^2 - 3/X^2$

$0 = \dfrac{3X^4 - 3}{X^2}$

Y' DNE AT $X = 0$
$Y' = 0$ AT $X = \pm 1$

INTERVAL	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
TEST	-2	$-1/2$	$1/2$	2
SIGN Y'	$+$	$-$	$-$	$+$
BEHAVIOR OF Y	↗	↓	↓	↗

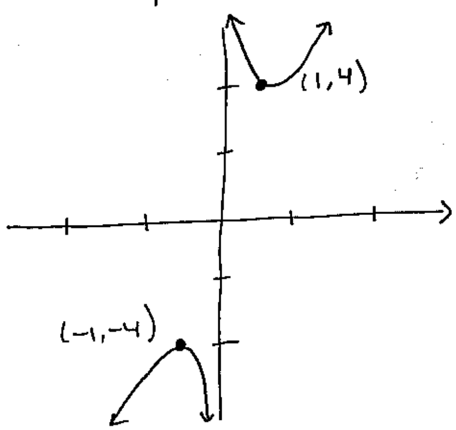
MAX AT $X = -1 \quad Y = -4 \quad (-1, -4)$
MIN AT $X = 1 \quad Y = 4 \quad (1, 4)$

⑥ $Y'' = 6X + 6/X^3 = \dfrac{6X^4 + 6}{X^3}$

Y'' DNE AT $X = 0$

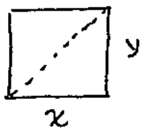
INTERVAL	$(-\infty, 0)$	$(0, \infty)$
TEST	-1	
SIGN Y''	$-$	$+$
CONCAVITY	∩	∪

NO INFL PT. AT $X = 0$



OPTIMIZATION p. 724-25

#13



$$D = \sqrt{x^2 + y^2}$$

$$2x + 2y = 48$$
$$y = 24 - x$$

$$D' = \tfrac{1}{2}\left(x^2 + (24-x)^2\right)^{-1/2}\left(2x + 2(24-x)(-1)\right)$$

$$= \frac{2x - 48 + 2x}{2\sqrt{x^2 + (24-x)^2}}$$

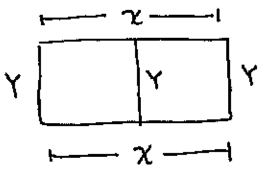
$$= \frac{4(x-12)}{2\sqrt{x^2 + (24-x)^2}}$$

$D' = 0$ WHEN $x = 12$

INTERVAL	$(-\infty, 12)$	$(12, \infty)$
TEST	0	13
SIGN	$-$	$+$
	$\searrow$	$\nearrow$

MIN AT $(12 = x)$

#16



$$2x + 3y = 240$$
$$y = \frac{240 - 2x}{3}$$

$$A = xy$$
$$= x\left(\frac{240 - 2x}{3}\right)$$
$$= \frac{240}{3}x - \frac{2x^2}{3}$$

$$A' = \frac{240}{3} - \frac{4x}{3}$$

$A' = 0$

WHEN

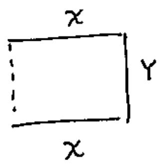
$$\frac{4x}{3} = \frac{240}{3}$$

$$x = 60$$

INTERVAL	$(-\infty, 60)$	$(60, \infty)$
TEST	0	100
SIGN OF A'	$+$	$-$
BEHAVIOR OF A	$\nearrow$	$\searrow$

MAX AT $x = 60$ $y = 40$

#18



$2x + Y = 800$

$Y = 800 - 2x$

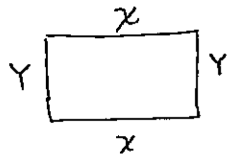
$A = xY$
$= x(800 - 2x)$
$= 800x - 2x^2$

$A' = 800 - 4x \qquad A' = 0 \qquad 800 = 4x$
$x = 200$

Interval	$(-\infty, 200)$	$(200, \infty)$
Test	0	300
sign A'	+	—
Behavior of A	↗	↘

MAX AT $X = 200$
$Y = 400$

#20



$A = xY = 1350$

$Y = 1350/x$

$\text{COST} = 2Y + x + 2x$
$C = 2Y + 3x$
$C = 2\left(\frac{1350}{x}\right) + 3x$

$C' = -\frac{2700}{x^2} + 3$

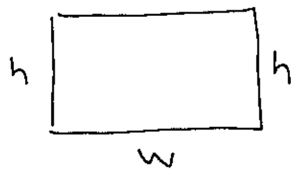
$C' = 0 \qquad -2700 = -3x^2$
$x^2 = 900$
$x = \pm 30$

C' DNE AT $x = 0$

Intervals	$(0, 30)$	$(30, \infty)$
Test	1	40
sign of C'	—	+
	↘	↗

MIN COST AT $X = 30$
$Y = 45$

#24



$2h + w = 36 \qquad w = 36 - 2h$

$A = hw$
$= h(36 - 2h) = 36h - 2h^2$

$A' = 36 - 4h$

$A' = 0$ WHEN $h = 9$

Interval	$(-\infty, 9)$	$(9, \infty)$
Test	0	10
sign A'	+	—
Behavior A	↗	↘

MAX AT $\left(\begin{array}{c} h = 9 \\ w = 18 \end{array} \right)$.