SOLUTION TO
BONUS EXERCISE
IMPLICIT DIFFERENTIATION
NYA-ELECTRO

Given $\ln(\sin(xy)) = -xy^3$, find y'.

SOLUTION 1

$$\frac{1}{\sin xy}\cos xy\,(y+xy') = -y^3 - x(3y^2)y'$$

$$\cos xy\,(y+xy') = -(\sin xy)y^3 - 3xy^2y'(\sin xy)$$

$$y\cos xy + xy'\cos xy = -y^3\sin xy - 3xy^2y'\sin xy$$

$$xy'\cos xy + 3xy^2y'\sin xy = -y^3\sin xy - y\cos xy$$

$$y'(x\cos xy + 3xy^2\sin xy) = -y^3\sin xy - y\cos xy$$

$$\boxed{y' = \frac{-y^3\sin xy - y\cos xy}{x\cos xy + 3xy^2\sin xy}}$$

SOLUTION 2

$$\frac{1}{\sin xy}\cos xy(y+xy') = -y^3 - 3xy^2y'$$

$$\cot xy\,(y+xy') = -y^3 - 3xy^2y'$$

$$y\cot xy + xy'\cot xy = -y^3 - 3xy^2y'$$

$$xy'\cot xy + 3xy^2y' = -y^3 - y\cot xy$$

$$y'(x\cot xy + 3xy^2) = -y^3 - y\cot xy$$

$$\boxed{y' = \frac{-y^3 - y\cot xy}{x\cot xy + 3xy^2}}$$

SOLUTION 3

$$\frac{1}{\sin xy}\cos xy(y+xy') = -y^3 - 3xy^2y'$$

$$y + xy' = \tan xy(-y^3 - 3xy^2y')$$

$$xy' + 3xy^2y'\tan xy = -y - y^3\tan xy$$

$$y'(x + 3xy^2\tan xy) = -y - y^3\tan xy$$

$$\boxed{y' = \frac{-y - y^3\tan xy}{x + 3xy^2\tan xy}}$$