Dawson College
Calculus II
201-NYB-05 Section 2
Winter 2010

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Website: http://www.obeymath.org
The website contains the solutions to the quizzes, tests and additional examples. It also has the material of previously taught courses.

Term Work: *(possibly worth 50% of final grade, see Grading Policy)*:

3 Class Tests*	worth a total of 40%	on:
	Test 1	**Friday February 26th**
	Test 2	**Friday April 9th**
	Test 3	**Friday May 7th**
Quizzes**	worth a total of 10%	on:
		every Friday except on test days

* Each class test is an hour and half long.
** Each quiz is 15 minutes long. The contents of the quiz is taken directly from the assigned excercises of previous lectures.

Important: There will be no make-up tests or quizzes. If a valid medical note is presented the weight of the quiz or test will be transferred to the weight of the final examination.

Winter 2010

DAWSON COLLEGE
MATHEMATICS DEPARTMENT
STUDY GUIDE
201-NYB-05 (CALCULUS II) – SCIENCE
PONDÉRATION 3-2-3

Prerequisite: Calculus I (201-NYA/201-NYA-05) preferably the Science version.

Objectives and Standards: This course introduces the student to Integral Calculus, to the techniques of integration and to some of the applications of integration to physical problems. Another look at limits and an introduction to the topic of infinite series are included. Use of mathematical software will be explored. For more details, see pages 44 to 49 of the Dawson Science Program.

Text: **Single Variable Essential Calculus – Early Transcendals by James Stewart**

Reference Books:
Calculus of a Single Variable – (8^{th} edition), by Larson, Hostetler and Edwards
Calculus Single Variable – 5^{th} edition by James Stewart
Calculus by Edwards & Penny,
Or any standard text book on Calculus of a Single Variable.

Methodology: Most sections use lectures and problem sessions. Some sections may use web-based assignments.

Termwork: The term grade is based on a minimum of 4½ hours of tests/quizzes.

Final Examination: The Final Examination will be a supervised, comprehensive examination held during the formal examination period. There are no exemptions.

Grading Policy: The grade shall consist of the greater of:
(A) Termwork for 50% and Final Exam for 50%
OR
(B) Final Exam for 100%.
To qualify for (B) the student must have obtained at least 50% of the term mark and must have written more than 50% of the class tests.

Calculators: A calculator without text storage or graphing capabilities is allowed for class test and final exam.

Literacy Policy: "Problem-solving is an essential component of this course. Students will be expected to analyze problems stated in words, to present their solutions logically and coherently, and to display their answers in a form corresponding to the statement of the problem, including appropriate units of measurement. Marks will be deducted for work which is inadequate in these respects, even though the answers may be numerically correct."

Formula Sheets: No formula sheet will be permitted for quizzes, class tests and the final exam.

Standard of Performance: In order to pass this course the student must obtain a final grade of at least 60%.

Department Website:
For final examinations from previous years and other useful information consult the departmental website. Go to www.dawsoncollege.qc.ca ⟶ go to Departments ⟶ go to Mathematics ⟶ go to Departmental Website ⟶ go to Help For Students.

Religious Holidays:

Students who wish to observe religious holidays must inform each of their teachers in writing within the first two weeks of each semester of their intent to observe the holiday so that alternative arrangements convenient to both the student and the teacher can be made at the earliest opportunity. The written notice must be given even when the exact date of the holiday is not known until later. Students who make such arrangements will not be required to attend classes or take examinations on the designated days, nor be penalized for their absence. It must be emphasized, however, that this College policy should not be interpreted to mean that a student can receive credit for work not performed. It is the student's responsibility to fulfill the requirements of the alternative arrangement.

Policy on Cheating and Plagiarism

Cheating in Examinations, Tests, and Quizzes

Cheating includes any dishonest or deceptive practice relative to formal final examinations, in-class tests, or quizzes. Such cheating is discoverable during or after the exercise in the evaluation process by the instructor. Such cheating includes, but is not limited to

a. copying or attempting to copy another's work.
b. obtaining or attempting to obtain unauthorized assistance of any kind.
c. providing or attempting to provide unauthorized assistance of any kind.
d. using or possessing any unauthorized material or instruments which can be used as information storage and retrieval devices.
e. taking an examination, test, or quiz for someone else.
f. having someone take an examination, test, or quiz in one's place.

Unauthorized Communication

Unauthorized communication of any kind during an examination, test, or quiz is forbidden and subject to the same penalties as cheating.

Plagiarism on Assignments and the Comprehensive Assessment

Plagiarism is the presentation or submission by a student of another person's assignments or Comprehensive Assessment as his or her own. Students who permit their work to be copied are considered to be as guilty as the plagiarizer.

Obligation of the Teacher

Every instance of cheating or plagiarism leading to a resolution that impacts on a student's grade must be reported by the teacher, with explanation, in writing to the Chair of Mathematics and to the Dean of Pre-University Studies. A copy of this report must also be given to the student.

Penalties

Cheating and plagiarism are considered extremely serious academic offences. Action in response to an incident of cheating and plagiarism is within the authority of the teacher.
Penalties may range from zero on a test, to failure of the course, to suspension or expulsion from the college.

Students' Obligations:

(a) Students have an obligation to remain informed about what takes place in their regularly scheduled classes. Absence from class does not excuse students from this responsibility.
(b) Students have an obligation to arrive on time and remain for the duration of scheduled classes and activities.
(c) Students have an obligation to write tests and final examinations at the times scheduled by the teacher or the College. Students have an obligation to inform themselves of, and respect, College examination procedures.
(d) Students have an obligation to show respectful behavior and appropriate classroom deportment. Should a student be disruptive and/or disrespectful, the teacher has the right to exclude the disruptive student from learning activities (classes) and may refer the case to the Director of Student Services under the Student Code of Conduct.
(e) Cellular phones, pagers and musical listening devices have the effect of disturbing the teacher and other students. All these devices should be turned off. Students who do not observe these rules will be asked to leave the classroom.
(f) Cell phones must also be put away. Text messaging is not allowed in class.

Specific Competencies	Learning Activities	Chapters, Sections & Problems in Text
Antiderivatives Reimann Sums – The Fundamental Theorem of Calculus	Find the area under a simple curve using Riemann sums. Define the Definite Integral as the limit of a Riemann sum. Prove the Fundamental theorem of Calculus, Substitution Method, Average Value.	4.7 p246, 1-30; 33-41 Review p250, 49-56 5.1 p260, 1, 3, 5; 11-14 5.2 p272, 1, 3, 5; 11-26; 29 5.3 p282, 1-27; 41-46 5.4 p291, 1-20 5.5 p299, 1-54 Review p302, 7-32; 35-38, 42, 46 and handout of extra problems on Average Value
Techniques of Integration	Integrate certain functions using the following techniques: - Integration by Parts - Trigonometric Integral using Identities - Trigonometric Substitutions - Partial Fractions (linear, repeated linear and quadratic factors only)	6.1 p309, 1-31 6.2 p319, 1-62 6.3 p327, 1-31; 35-42 Review p355, 1-40
Numerical Integration	Approximate certain integrals using Simpson's Rule	6.5 p343, 7-16 Review p356, 57, 58
Indeterminate Forms and L'Hopitals Rule	Evaluate Limits of Indeterminate Forms using L'Hopital's Rule	3.7 p193, 1-36 Review p197, 67-82
Improper Integrals	Determine the Convergence of Improper Integrals	6.6 p352, 5-32 Review p355, 41-50
Applications of Integration	Extend the notion of the Definite Integral to calculate: - the area bounded between two curves - the volume of a solid of revolution using both the disk and shell methods - arc length (to follow Techniques of Integration) - the work done in pumping fluid	7.1 p361, 1-16 7.2 p370, 1-16 7.3 p376, 1-26 7.4 p383, 1-10; 12-14 7.5 p394, 1-3, 6-8, 17, 18 and handout of extra problems on work. Review p408, 1-14, 25, 26, 31, 37
Infinite Sequence	Examine the convergence or divergence of Infinite Sequences	8.1 p419, 1-23; 26, 27
Infinite Series	Determine the sum of an infinite series from the definition. Determine the convergence of a geometric series; applications Tests for convergence of series of positive terms; Integral, Comparison, Limit Comparison and Ratio tests.	8.2 p427, 1-26, 31, 32 8.3 p436, 3, 4, 6-26, 27 8.4 p446, 19-31, 36, 37, 39 Review p480, 1-18, 25-29
MacLaurin and Taylor's Polynomials	MacLaurin and Taylor's Polynomials and approximations	8.8 p477, 3-8, 9-13 (Part a only) Review p480, 43-50 (MacLaurin Polynomial only)

AVERAGE VALUE - MEAN VALUE THEOREM for INTEGRALS

The Mean Value Theorem for Integrals

If f is continuous on the closed interval [a, b] there exists c in [a, b] such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

f(c) is called the average value of f on [a, b].

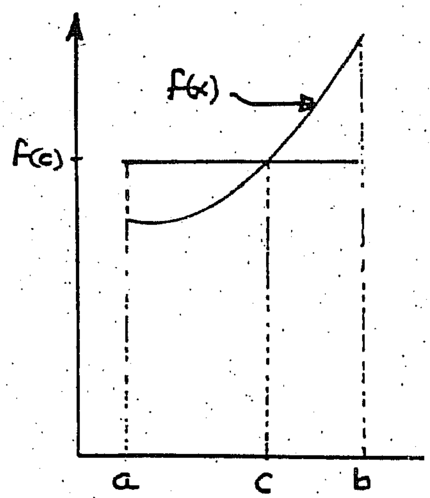


Figure 1 shows the average value f(c). Another way to view the Mean Value Theorem is that the product f(c)(b − a) gives the area of a rectangle with height f(c) and base b − a. This area corresponds to the area under the curve.

EXAMPLE 1 Calculate the average value of a) $f(x) = 4 - x^2$ on [−2, 2]
b) $f(x) = x\sqrt{4 - x^2}$ on [0, 2]

SOLUTION: a) $f(c) = \frac{1}{2-(-2)}\int_{-2}^{2}(4 - x^2)\,dx = \frac{1}{4}\left(4x - \frac{1}{3}x^3\right)\Big|_{-2}^{2}$

$$= \frac{1}{4}\left(8 - \frac{8}{3} - (-8 - \left(\frac{-8}{3}\right)\right) = \frac{1}{4}\left(\frac{16}{3} - \left(-\frac{16}{3}\right)\right) = \frac{8}{3}$$

b) $f(c) = \frac{1}{2-0}\int_0^2 x\sqrt{4 - x^2}\,dx$ Let $u = 4 - x^2$, $du = -2x\,dx$, $-\frac{du}{2} = x\,dx$
$x = 0, u = 4;\ x = 2, u = 0$

$$= \frac{1}{2}\int_4^0 \sqrt{u}\left(\frac{-du}{2}\right)$$

$$= \left(\frac{-1}{4}\right)\left(\frac{2}{3}\right)u^{3/2}\Big|_4^0 = \left(\frac{-1}{6}\right)\left\{0 - 4^{3/2}\right\}$$

$$= \left(\frac{-1}{6}\right)(-8) = \frac{4}{3}$$

EXAMPLE 2 The Dawson College Physics Department held a competition for students to design a vehicle powered only by gravity. The velocity of one of the vehicles entered was modelled by the function $v = \frac{3t^2}{\sqrt{t^3 + 36}}$ m/s (meters/second). Calculate the distance covered and the average velocity from t = 0 to t = 4 seconds.

SOLUTION: Since velocity is the derivative of distance $\left(v = \frac{ds}{dt}\right.$ where s is the distance in meters$\left.\right)$ then distance is the integral of velocity. The net distance travelled between two points in time t_1 and t_2 can be calculated by the definite integral

$$s = \int_{t_1}^{t_2} v\, dt = \int_0^4 \frac{3t^2}{\sqrt{t^3+36}}\, dt \qquad \text{Let } u = t^3 + 36,\ du = 3t^2\, dt$$

$$t = 0, u = 36; \quad t = 4, u = 100$$

$$= \int_{36}^{100} \frac{du}{\sqrt{u}} = \int_{36}^{100} u^{-1/2}\, du = \left(2\sqrt{u}\right)\Big|_{36}^{100} = 2\left(\sqrt{100} - \sqrt{36}\right) = 2(10-6) = 8 \text{ m/s.}$$

The average velocity is calculated by dividing this integral by the time interval which is 4 seconds. Hence $v_{avg} = 8/4 = 2$ m/s. This corresponds to our usual notion of average velocity which is defined as the distance travelled divided by the time interval.

<u>EXAMPLE 2</u> One day in Montreal, snow began falling at 10 A.M. The rate of accumulation was given by $\frac{dA}{dt} = \frac{3}{4}\sqrt{t} + \frac{\pi}{2}\sin\left(\frac{\pi}{8}t\right)$ cm/hr. where t is the time in hours after 10 A.M. What was the total accumulation A and the average rate of accumulation from 10 A.M. to 2 P.M.?

SOLUTION: $A = \int_0^4 \left(\frac{3}{4}\sqrt{t} + \frac{\pi}{2}\sin\left(\frac{\pi}{8}t\right)\right)dt = \frac{3}{4}\left(\frac{2}{3}\right)t^{3/2} - \frac{\pi}{2}\left(\frac{8}{\pi}\right)\cos\left(\frac{\pi}{8}t\right)\Big|_0^4$

$$= \frac{1}{2}(4)^{3/2} - 4\cos\left(\frac{\pi}{2}\right) - \left(0 - 4\cos 0\right) = 4 - 0 + 4 = 8 \text{ cm.}$$

$$\frac{dA}{dt}_{avg} = \frac{8}{4} = 2 \text{ cm/hr.}$$

<u>EXAMPLE 3</u> The power output of a solar cell is given by $p = 50\sin^2\left(\frac{\pi}{12}t\right)$ joules/hr. where t is the time in hours after 6 A.M. How many joules of energy are produced fron 6 A. M. to 6 P.M. and what is the average power produced in this time period. Take $t = 0$ as 6 A. M. Hint: To integrate $\sin^2\theta$ we need the trig identity $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

SOLUTION: $p = \frac{dE}{dt}$ where E is the energy in joules. Hence the energy produced from $t = 0$ to $t = 12$ hours is

$$E = \int_0^{12} 50\sin^2\left(\frac{\pi}{12}t\right) dt = \int_0^{12} 50\left(\frac{1-\cos\left(\frac{\pi}{6}t\right)}{2}\right) dt = 25\left(t - \frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right)\Big|_0^{12}$$

$$= 25\left(12 - \frac{6}{\pi}(\sin 2\pi - \sin 0)\right) = 25(12) = 300 \text{ joules.} \quad p_{avg} = \frac{300}{12} = 25 \text{ joules/hr.}$$

Note that the average power is exactly half the peak power.

ASSIGNMENT - AVERAGE VALUE

1) Find the average value of the function on the interval indicated. Find all values of x where the function equals its average value.

a) $f(x) = \frac{x^2 + 1}{x^2}$: $[\frac{1}{2}, 2]$ b) $f(x) = \frac{x}{\sqrt{x^2 + 9}}$: $[0, 4]$

2) The velocity of a rocket is given by $v = t^2\sqrt{t^3 + 64}$ m./sec. What is the distance travelled and the average velocity from $t = 0$ to $t = 8$ sec.? ($v = \frac{ds}{dt}$)

3) On a certain day the rate of accumulation of snowfall after 7:00 P. M. ($t = 0$) was given by $\frac{dA}{dt} = 2t + \pi\cos(\frac{\pi}{4}t)$ cm./ hr. What is the total accumulation, A, of snowfall and the average rate of snowfall by 9:00 P. M.?

4) The power used by an electric motor is given by $p = 15 + 2\sin(\frac{\pi}{24}t)$ joules/hr. where t is the time in hours after midnight. How much energy is consumed in one days operation and what is the average power?

ANSWERS

1) a) 2, $x=1$ b) 1/2, $x=\sqrt{3}$

2) $s=2958$ m., $v_{avg}=369.8$ m./sec.

3) $A=8$ cm, $\frac{dA}{dt}_{avg}=4$ cm./hr.

4) $E=390.6$ joules, $p_{avg}=16.27$ joules/hr.

PROBLEMS ON WORK

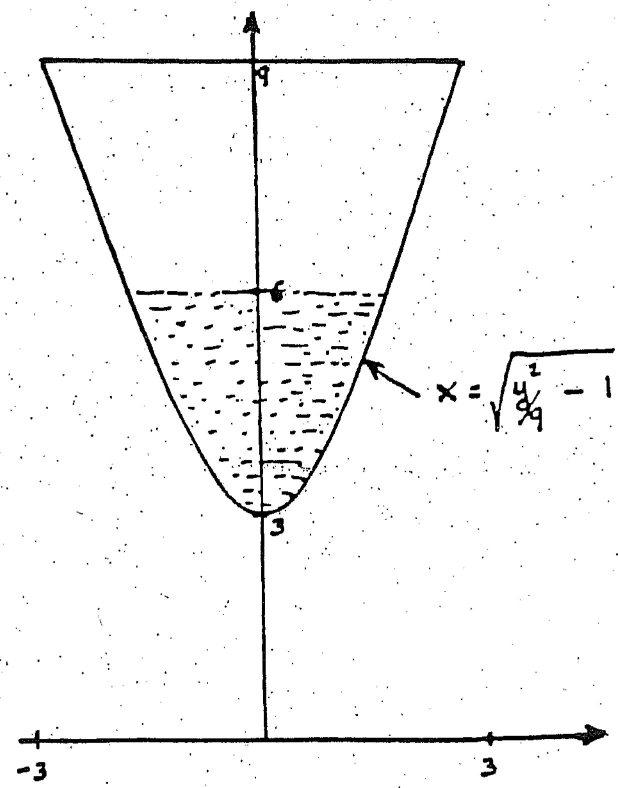
1. a) How much work is required to put a 5 ton satellite in an orbit 150 miles above the surface of Earth.
   (Take the radius of Earth as 4 000 miles and give your answer in ton-miles.)

   b) How much additional work would be required to put the satellite in geosynchronous orbit (orbits the Earth once a day) at an altitude of 22 400 miles above the surface of Earth.

2. A tank is located with its base 3 m. above ground level. The shape of the tank is formed by revolving the hyperbola $\frac{y^2}{9} - x^2 = 1$ about the y axis for $3 \leq y \leq 9$.

   a) Calculate the work done to fill the tank to a depth of 3 m. with oil of density 800 kg/m^3 if the oil is pumped in from ground level.

   b) Now calculate the work to pump all the oil up over the top of the tank.

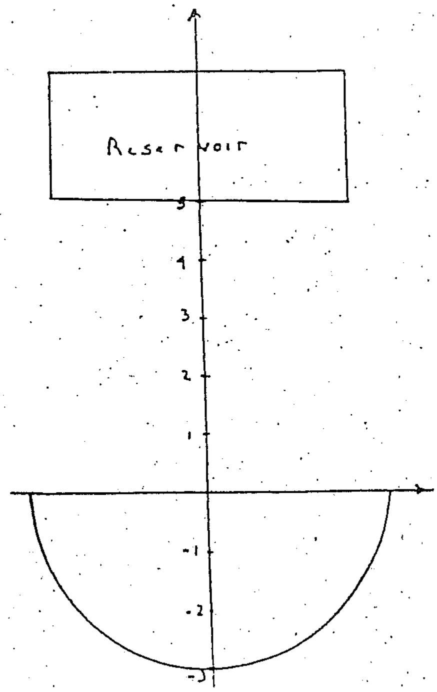


3. A tank is located with its base at ground level. The tank is 9 ft. high and 12 ft. in diameter on top. It is formed by revolving the parabola $y = \frac{x^2}{4}$ about the y axis for $0 \leq y \leq 9$. Calculate

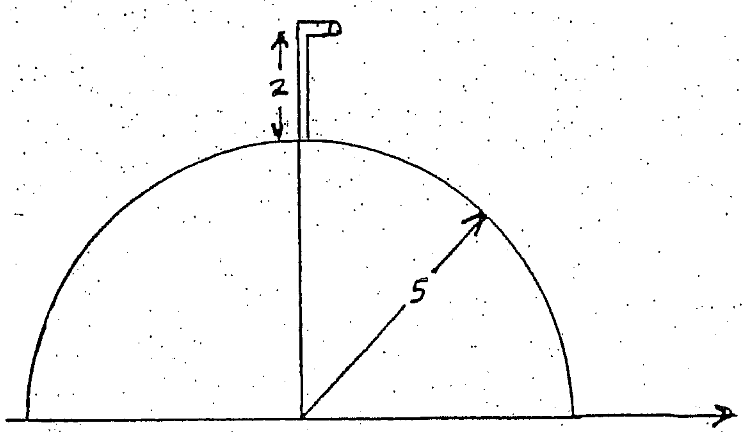
   a) the work required to fill this tank to a depth of 4 ft. with oil of density 50 lb/ft^3 if the oil is pumped in from a reservoir located 6 ft. below ground level.

   b) the work required to pump all the oil up over the top of the tank.

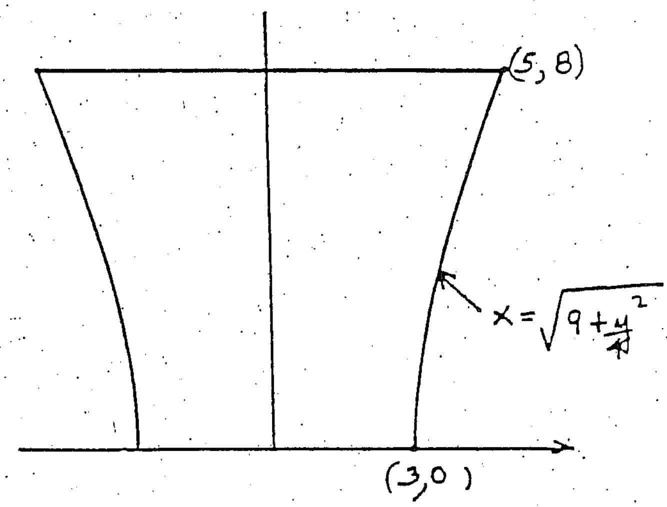
4. A tank in the shape of a hemisphere has a radius of 3 m. and is filled to the top with glycol of density 1113 kg/m^3. Calculate the work required to pump all the glycol to a reservoir 5 m. above the top of the tank.



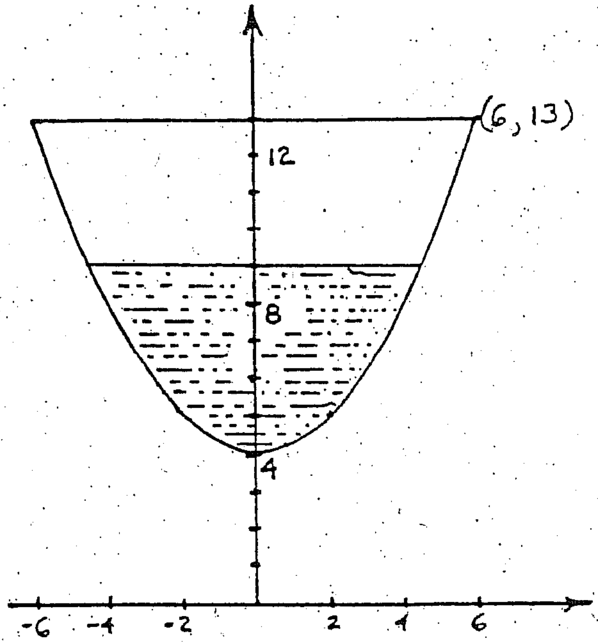
5. A tank has the shape of the top half of a sphere of radius 5 meters. It is filled with oil of density 900 kg/m^3. Find the work required to empty the tank by pumping the oil to an outlet 2 meters above the top of the tank.

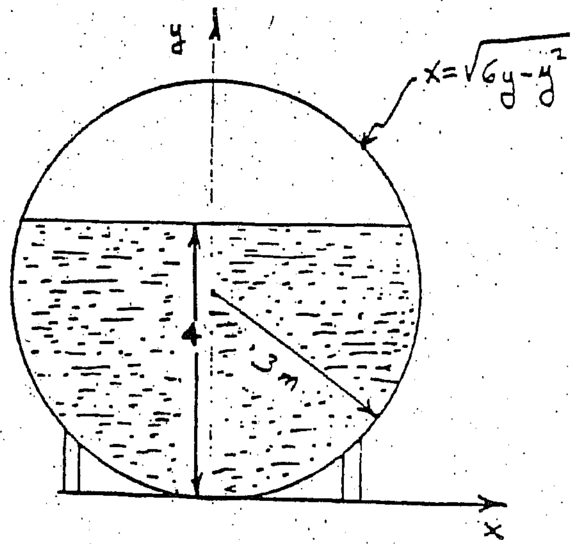


6. A storage tank of height 8 m. has a shape which is generated by revolving the hyperbola $x^2 = 9 + \frac{y^2}{4}$ about the y axis. If the tank is filled with oil of density 800 kg/m^3 to a depth of 4 m., calculate the work required to pump all the oil over the top of the tank.



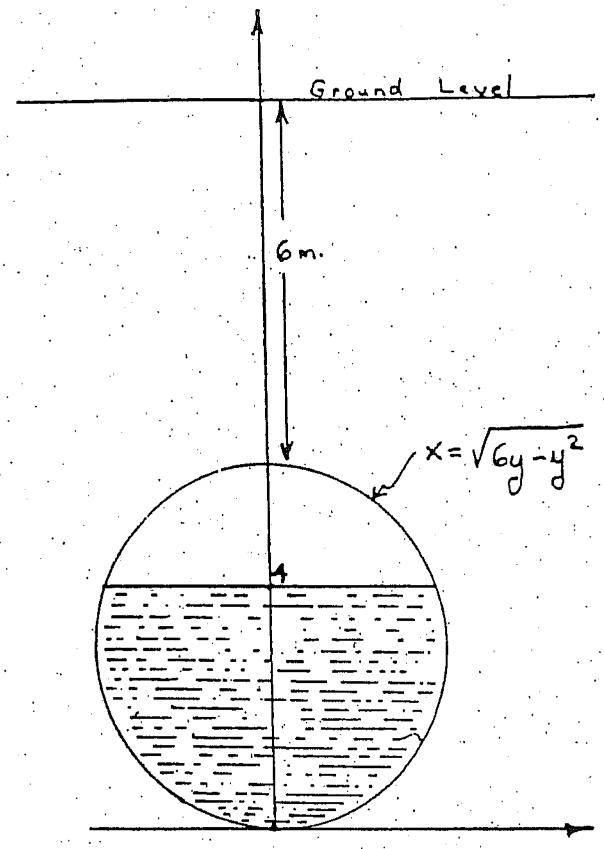
7. A tank which is 9 ft. high, has its base 4 ft. above ground level and is formed by revolving the parabola $y = \frac{x^2}{4} + 4$ about the y axis. If the tank is filled to a depth of 5 ft. with oil of density 50 lb/ft^3, find the work done in pumping all the oil up over the top of the tank.



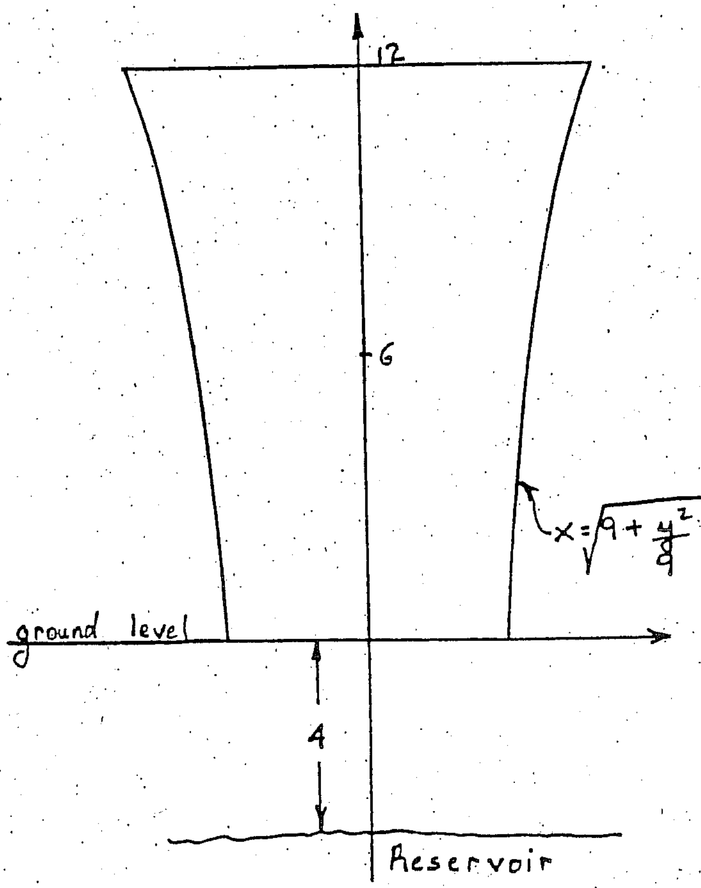


8. A Horton Sphere of radius 3 m. is located with its base on the ground. If we locate this spherical tank with its base at the origin, the equation of the cross section in the 1st. quadrant is $x = \sqrt{6y - y^2}$. The tank is filled with oil of density 800 kg/m^3 to a depth of 4 m. Calculate the work required to pump all the oil up over the top of the tank.

9. An underground storage tank in the shape of a sphere of radius 3 m. is located with its top 6 m. below ground level. The equation for the sphere is $x^2 + y^2 - 6y = 0$ with the origin located at the bottom of the tank. If the tank is filled with gasoline of density $\rho = 940$ kg/m^3 to a depth of 4 m., calculate the work required to pump all the gasoline up to ground level.



10. A storage tank of height 12 m. has a shape which is generated by revolving the hyperbola $x^2 - \frac{y^2}{9} = 9$ about the y axis. A large reservoir containing oil of density 840 kg./m^3 is located 4 m. below ground level. Calculate the work required to fill the tank to a depth of 9 m. by pumping oil in through the bottom of the tank. You may assume that the oil in the reservoir remains at a level 4 m. below the bottom.



ANSWERS

A Note Concerning Densities.

In the Metric System the mass density ρ is usually given in kg/m^3. This must be converted to the weight density δ by multiplying by gravity $g = 9.8\ m/s^2$ so that $\delta = g\rho$. In the Imperial System no conversion is required. The weight density δ is given in pounds/foot^3 (lb./ft.^{3})

1) $F = 5 = \frac{k}{r^2}$ so $k = 5(4000)^2 = 8 \times 10^7$

a) $W = \int_{4000}^{4150} \frac{8 \times 10^7}{r^2}\, dr = 723$ ton-miles

b) $W = \int_{4150}^{26400} \frac{8 \times 10^7}{r^2}\, dr = 1.625 \times 10^4$ ton-miiles

2) a) $W = \int_{3}^{6} \delta\pi(\frac{y^2}{9} - 1)y\, dy = 4.988 \times 10^5$ joules.

b) $W = \int_{3}^{6} \delta\pi(\frac{y^2}{9} - 1)(9 - y)\, dy = 3.879 \times 10^5$ j.

3) a) $W = \int_{0}^{4} \delta\pi(4y)(6 + y)\, dy = 4.356 \times 10^4$ ft.-lb.

b) $W = \int_{0}^{4} \delta\pi(4y)(9 - y)\, dy = 3.183 \times 10^4$ ft.-lb.

4) $W = \int_{-3}^{0} \delta\pi(9 - y^2)(5 - y)\, dy = 3.778 \times 10^6$ j.

5) $W = \int_{0}^{5} \delta\pi(25 - y^2)(7 - y)\, dy = 1.183 \times 10^7$ j.

6) $W = \int_0^4 \delta\pi(9 + \frac{y^2}{4})(8 - y)\,dy = 5.977 \times 10^6$ j.

7) $W = \int_4^9 \delta\pi 4(y - 4)(13 - y)\,dy = 4.451 \times 10^4$ ft.-lb.

8) $W = \int_0^4 \delta\pi(6y - y^2)(6 - y)\,dy = 2.364 \times 10^6$ j.

9) $W = \int_0^4 \delta\pi(6y - y^2)(12 - y)\,dy = 7.409 \times 10^6$ j.

10) $W = \int_0^9 \delta\pi(9 + \frac{y^2}{4})(y + 4)\,dy = 2.531 \times 10^7$ j.