

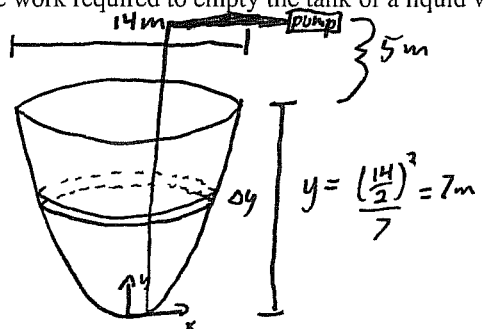
## Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (5 marks) §7.4 #5 Find the length of the curve:  $y = \frac{x^5}{6} + \frac{1}{10x^3}$ ,  $1 \leq x \leq 2$ .

$$\begin{aligned}
 s &= \int_1^2 \sqrt{1 + (y')^2} dx & y' &= \frac{5x^4}{6} - \frac{3}{10x^4} \\
 &= \int_1^2 \sqrt{1 + \left(\frac{5x^4}{6} - \frac{3}{10x^4}\right)^2} dx & &= \left[ \frac{x^5}{6} - \frac{1}{10x^3} \right]_1^2 \\
 &= \int_1^2 \sqrt{1 + \frac{25x^8}{36} - \frac{1}{2} + \frac{9}{100x^8}} dx & &= \frac{2^5}{6} - \frac{1}{10 \cdot 2^3} - \frac{1}{6} + \frac{1}{10} \\
 &= \int_1^2 \sqrt{\frac{25x^8}{36} + \frac{1}{2} + \frac{9}{100x^8}} dx & &= \frac{16}{3} - \frac{1}{80} - \frac{1}{6} + \frac{1}{10} = \frac{1261}{240} \\
 &= \int_1^2 \sqrt{\left(\frac{5x^4}{6} + \frac{3}{10x^4}\right)^2} dx \\
 &= \int_1^2 \left(\frac{5x^4}{6} + \frac{3}{10x^4}\right) dx
 \end{aligned}$$

**Question 2.** (5 marks) A tank has a 14m diameter on top, its shape is defined by revolving  $y = \frac{x^2}{7}$  about the y-axis. Set up the integral to find the work required to empty the tank of a liquid with an arbitrary density  $\rho$  from a pump 5m above the top of the tank.



force:  $\Delta F = \Delta mg$   
 $= 7\pi r y g \Delta y$

distance of slice to pump:  $12 - y$   
 $\Delta W = F \cdot d$   
 $\therefore \Delta W = 7\pi r y g (12 - y) \Delta y$

$\therefore W = \int_0^7 7\pi r y g (12 - y) dy$

Volume:  $\Delta V = \pi r^2 \Delta y$   
 $= \pi x^2 \Delta y$   
 $= \pi 7y \Delta y$

mass:  $\Delta m = \Delta V \rho$   
 $= 7\pi r y \Delta y$