**Dawson College: Calculus II**: 201-NYB-05-S2: Winter 2010

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# Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** *(5 marks)* §7.4 #6 Find the length of the curve: $y = \frac{x^2}{2} - \frac{\ln x}{4}$, $2 \le x \le 4$.

$$S = \int_2^4 \sqrt{1 + (y')^2}\, dx \qquad y' = \frac{2x}{2} - \frac{1}{4x}$$

$$= x - \frac{1}{4x}$$

$$= \int_2^4 \sqrt{1 + \left(x - \frac{1}{4x}\right)^2}\, dx$$

$$= \int_2^4 \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}}\, dx$$

$$= \int_2^4 \sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}}\, dx$$

$$= \int_2^4 \sqrt{\left(x + \frac{1}{4x}\right)^2}\, dx$$

$$= \int_2^4 x + \frac{1}{4x}\, dx$$

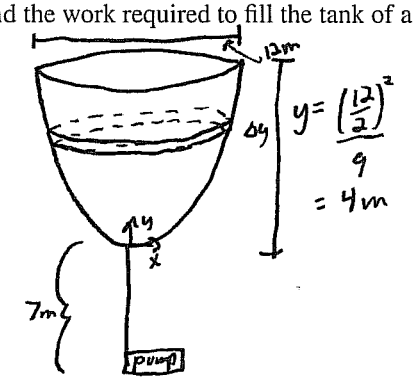
$$= \left[\frac{x^2}{2} + \frac{\ln|x|}{4}\right]_2^4$$

$$= \frac{4^2}{2} + \frac{\ln 4}{4} - \left[\frac{2^2}{2} + \frac{\ln 2}{4}\right]$$

$$= 8 + \frac{\ln 2}{2} - 2 - \frac{\ln 2}{4}$$

$$= 6 + \ln \sqrt[4]{2}$$

**Question 2.** *(5 marks)* A tank has a $12m$ diameter on top, its shape is defined by revolving $y = \frac{x^2}{9}$ about the y-axis. Set up the integral to find the work required to fill the tank of a liquid with an arbitrary density $\rho$ from a pump $7m$ below the tank.




Volume: $\Delta V = \pi r^2 \Delta y$
$= \pi x^2 \Delta y$
$= \pi\, 9y\, \Delta y$

mass: $\Delta m = \Delta V \rho$
$= 9\pi\rho y\, \Delta y$

force: $\Delta F = \Delta m g$
$= 9\pi\rho g y \Delta y$

distance of slice to pump: $7 + y$

$\therefore \Delta W = \Delta F \cdot d$
$= 9\pi\rho g y (7+y) \Delta y$

$\therefore W = \int_0^4 9\pi\rho g y (7+y)\, dy$