Dawson College: Calculus II: 201-NYB-05-S2: Winter 2010

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Quiz 11

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.1 #21 Determine whether the sequence converges ot diverges. If it converges, find the limit.

$a_n = \frac{\cos^2 n}{2^n}$

$b_n = 0 \leq a_n \leq \frac{1}{2^n} = c_n$ and

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} c_n = 0$ then by the squeeze

theorem $\lim_{n\to\infty} a_n = 0$.

Question 2. *(5 marks)* §8.2 #13 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty} \frac{1}{3^n} + \sum_{n=1}^{\infty} \frac{2^n}{3^n}$$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n + \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n + a_0 - a_0 + \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n + b_0 - b_0$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n - a_0 + \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n - b_0$$

$$= \frac{1}{1-1/3} - \left(\frac{1}{3}\right)^0 + \frac{1}{1-2/3} - \left(\frac{2}{3}\right)^0$$

$$= \frac{3}{2} - 1 + 3 - 1 = \frac{5}{2}$$

Question 3. *(5 marks)* §8.2 #19 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1}$$

$$\frac{2}{n^2-1} = \frac{2}{(n+1)(n-1)} = \frac{A}{n+1} + \frac{B}{n-1}$$

$$2 = A(n-1) + B(n+1)$$

Let $n=1$

$$2 = A(1-1) + B(1+1)$$
$$1 = B$$

Let $n=-1$

$$2 = A(-1-1) + B(-1+1)$$
$$2 = -2A$$
$$-1 = A$$

$$\therefore \sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty} \left[\frac{1}{n-1} - \frac{1}{n+1}\right]$$

$$S_n = a_2 + a_3 + a_4 + a_5 + a_6 + \cdots + a_{n-4} + a_{n-3} + a_{n-2} + a_{n-1} + a_n$$

$$= \left[\frac{1}{1} - \cancel{\frac{1}{3}}\right] + \left[\frac{1}{2} - \cancel{\frac{1}{4}}\right] + \left[\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right] + \left[\cancel{\frac{1}{4}} - \frac{1}{6}\right] + \left[\cancel{\frac{1}{5}} - \frac{1}{7}\right]$$

$$+ \cdots + \left[\frac{1}{n-5} - \cancel{\frac{1}{n-3}}\right] + \left[\frac{1}{n-4} - \cancel{\frac{1}{n-2}}\right] + \left[\cancel{\frac{1}{n-3}} - \cancel{\frac{1}{n-1}}\right] + \left[\cancel{\frac{1}{n-2}} - \frac{1}{n}\right]$$

$$+ \left[\cancel{\frac{1}{n-1}} - \frac{1}{n+1}\right]$$

$$= 1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[1 + \frac{1}{2} - \cancel{\frac{1}{n}}^{\,0} - \cancel{\frac{1}{n+1}}^{\,0}\right] = \frac{3}{2}$$