Dawson College: **Calculus II**: 201-NYB-05-S2: Winter 2010

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Quiz 11

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.1 #24 Determine whether the sequence converges ot diverges. If it converges, find the limit.

$$a_n = \frac{\sin 2n}{1+\sqrt{n}}$$

$b_n = \dfrac{-1}{1+\sqrt{n}} \leq a_n \leq \dfrac{1}{1+\sqrt{n}} = c_n$ and

$\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} c_n = 0$ then by the squeeze theorem

$\lim\limits_{n\to\infty} a_n = 0.$

Question 2. *(5 marks)* §8.2 #16 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty}[(0.8)^{n-1} - (0.3)^n] = \sum_{n=1}^{\infty}(0.8)^{n-1} - \sum_{n=1}^{\infty}(0.3)^n$$

$$= \sum_{n=1}^{\infty}(0.8)^n(0.8)^{-1} - \sum_{n=1}^{\infty}(0.3)^n$$

$$= \sum_{n=1}^{\infty}(0.8)^n(0.8)^{-1} + a_0 - a_0 - \left[\sum_{n=1}^{\infty}(0.3)^n + b_0 - b_0\right]$$

$$= \sum_{n=0}^{\infty}(0.8)^n(0.8)^{-1} - a_0 - \sum_{n=1}^{\infty}(0.3)^n + b_0$$

$$= \frac{(0.8)^{-1}}{1-0.8} - (0.8)^{-1} - \frac{1}{1-0.3} + 1$$

$$= \frac{\frac{5}{4}}{1-\frac{4}{5}} - \frac{5}{4} - \frac{1}{1-\frac{3}{10}} + 1 = \frac{25}{4} - \frac{5}{4} - \frac{10}{7} + 1 = \frac{32}{7}$$

Question 3. *(5 marks)* §8.2 #21 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

$$\frac{3}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3}$$

$$3 = A(n+3) + Bn$$

Let $n = 0$

$$3 = 3A$$
$$1 = A$$

Let $n = -3$

$$3 = A(-3+3) + B(-3)$$
$$-1 = B$$

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)} = \sum_{n=1}^{\infty} \left[\frac{1}{n} - \frac{1}{n+3}\right]$$

$$S_n = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + \dots + a_{n-6} + a_{n-5} + a_{n-4} + a_{n-3} + a_{n-2} + a_{n-1} + a_n$$

$$= \left[\frac{1}{1} - \cancel{\frac{1}{4}}\right] + \left[\frac{1}{2} - \cancel{\frac{1}{5}}\right] + \left[\frac{1}{3} - \cancel{\frac{1}{6}}\right] + \left[\cancel{\frac{1}{4}} - \cancel{\frac{1}{7}}\right] + \left[\cancel{\frac{1}{5}} - \frac{1}{8}\right] + \left[\cancel{\frac{1}{6}} - \frac{1}{9}\right] + \left[\cancel{\frac{1}{7}} - \frac{1}{10}\right]$$

$$+ \dots + \left[\frac{1}{n-6} - \cancel{\frac{1}{n-3}}\right] + \left[\frac{1}{n-5} - \cancel{\frac{1}{n-2}}\right] + \left[\frac{1}{n-4} - \cancel{\frac{1}{n-1}}\right] + \left[\cancel{\frac{1}{n-3}} - \cancel{\frac{1}{n}}\right]$$

$$+ \left[\cancel{\frac{1}{n-2}} - \frac{1}{n+1}\right] + \left[\cancel{\frac{1}{n-1}} - \frac{1}{n+2}\right] + \left[\cancel{\frac{1}{n}} - \frac{1}{n+3}\right]$$

$$= 1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}$$

$$S = \lim_{n \to \infty} \left[1 + \frac{1}{2} + \frac{1}{3} - \cancelto{0}{\frac{1}{n+1}} - \cancelto{0}{\frac{1}{n+2}} - \cancelto{0}{\frac{1}{n+3}}\right]$$

$$= \frac{11}{6}$$