

Quiz 12

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §8.8 #7 Find the Taylor Polynomial of order 3 of $f(x) = xe^{-2x}$ at $x = 0$.

$$f(x) = xe^{-2x}, f(0) = 0e^0 = 0$$

$$f'(x) = e^{-2x} + (-2)xe^{-2x}, f'(0) = e^0 - 2(0)e^0 = 1$$

$$f''(x) = -2e^{-2x} - 2e^{-2x} + 4xe^{-2x}, f''(0) = -2e^0 - 2e^0 + 4(0)e^0 = -4$$

$$f'''(x) = 4e^{-2x} + 4e^{-2x} + 4e^{-2x} - 8xe^{-2x}, f'''(0) = 4e^0 + 4e^0 + 4e^0 - 8 = 12$$

$$\begin{aligned} \text{So } P_3(x) &= f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} \\ &= 0 + (x-0) - \frac{4(x-0)^2}{2!} + \frac{12(x-0)^3}{3!} \\ &= x - 2x^2 + 2x^3 \end{aligned}$$

Question 2. (5 marks) Evaluate the following integral:

$$\int_1^{\sqrt{2}} \frac{4+2\sqrt{x^2-1}}{x\sqrt{x^2-1}} dx = \lim_{a \rightarrow 1^+} \int_a^{\sqrt{2}} \frac{4}{x\sqrt{x^2-1}} dx + \int_1^{\sqrt{2}} \frac{2\sqrt{x^2-1}}{x\sqrt{x^2-1}} dx$$

$$= \lim_{a \rightarrow 1^+} 4 \left[\operatorname{arcsec} x \right]_a^{\sqrt{2}} + 2 \left[\ln|x| \right]_1^{\sqrt{2}}$$

$$= \lim_{a \rightarrow 1^+} \left[4 \left[\operatorname{arcsec} \sqrt{2} \right] - 4 \left[\operatorname{arcsec} a \right] \right] + 2 \ln \sqrt{2} - 2 \ln 1$$

$$= 4 \left(\frac{\pi}{4} \right) + 2 \ln \sqrt{2}$$

$$= \pi + \ln 2$$