

Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (6 marks) §5.3 #19 Use the definition of the integral to evaluate the integral (use the limit process).

$$\int_{-1}^5 1+3x \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{5-(-1)}{n} = \frac{6}{n}$$

$$x_i = a + i \Delta x = -1 + \frac{6i}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(-1 + \frac{6i}{n}\right) \frac{6}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{6}{n} \sum_{i=1}^n \left[1 + 3\left(-1 + \frac{6i}{n}\right) \right]$$

$$= \lim_{n \rightarrow \infty} \frac{6}{n} \sum_{i=1}^n \left[-2 + \frac{18i}{n} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{6}{n} \left[\sum_{i=1}^n -2 + \frac{18}{n} \sum_{i=1}^n i \right]$$

$$= \lim_{n \rightarrow \infty} \frac{6}{n} \left[-2n + \frac{18}{n} \frac{n(n+1)}{2} \right]$$

$$= \lim_{n \rightarrow \infty} \left[-12 + \frac{54(n+1)}{n} \right]$$

$$= -12 + 54$$

$$= 42$$

Question 2. (4 marks) §5.3 #26 Evaluate the integral.

$$\int_0^{\pi/3} \frac{\sin \theta + \sin \theta \tan^2 \theta}{\sec^2 \theta} \, d\theta = \int_0^{\pi/3} \frac{\sin \theta (1 + \tan^2 \theta)}{\sec^2 \theta} \, d\theta$$

$$= \int_0^{\pi/3} \frac{\sin \theta \sec^2 \theta}{\sec^2 \theta} \, d\theta$$

$$= \int_0^{\pi/3} \sin \theta \, d\theta$$

$$= \left[-\cos \theta \right]_0^{\pi/3} = -\cos \frac{\pi}{3} - \left[-\cos \theta \right]$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

