Dawson College: Calculus II: 201-NYB-05-S2: Winter 2010

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Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(6 marks)* §5.3 #19 Use the definition of the integral to evaluate the integral *(use the limit process)*.

$$\int_0^2 2-x^2\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \qquad \text{where } \Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$$

$$x_i = a + i\Delta x = \frac{2i}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\frac{2}{n}$$

$$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n} f\left(\frac{2i}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left[2-\left(\frac{2i}{n}\right)^2\right]$$

$$= \lim_{n\to\infty}\frac{2}{n}\left[\sum_{i=1}^{n}2 - \sum_{i=1}^{n}\frac{4i^2}{n^2}\right]$$

$$= \lim_{n\to\infty}\frac{2}{\cancel{n}}\left[2\cancel{n} - \frac{4}{n^2}\,\frac{\cancel{n}(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty}\left[4 - \frac{8(2n^2+3n+1)}{6}\right] = 4-\frac{16}{6} = 4-\frac{8}{3} = \frac{12-8}{3} = \frac{4}{3}$$

Question 2. *(4 marks)* §5.3 #25 Evaluate the integral.

$$\int_0^{\pi/4}\frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\frac{\pi}{4}}\frac{1}{\cos^2\theta} + \frac{\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}}\,d\theta$$

$$= \int_0^{\pi/4}\sec^2\theta + 1\,d\theta$$

$$= \Big[\tan\theta + \theta\Big]_0^{\pi/4}$$

$$= \left[\tan\frac{\pi}{4} + \frac{\pi}{4}\right] - \left[\tan 0 - 0\right]$$

$$= 1+\frac{\pi}{4}$$

$$= \frac{4+\pi}{4}$$