Dawson College: **Calculus II**: 201-NYB-05-S2: Winter 2010

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Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §5.4 #13 Use the Second Fundamental Theorem of Calculus to find the derivative of the function.

$$g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du = \int_{2x}^{0} \frac{u^2-1}{u^2+1}\,du + \int_{0}^{3x} \frac{u^2-1}{u^2+1}\,du$$

$$= -\int_{0}^{2x} \frac{u^2-1}{u^2+1}\,du + \int_{0}^{3x} \frac{u^2-1}{u^2+1}\,du$$

$$= -f(g_1(x)) + f(g_2(x)) \quad \text{where} \quad f(x) = \int_0^x \frac{u^2-1}{u^2+1}\,du$$

$g_1(x) = 2x$
$g_2(x) = 3x$

$$\therefore g'(x) = -f'(g_1(x))g_1'(x) + f'(g_2(x))g_2'(x)$$

$$= -\frac{(2x)^2-1}{(2x)^2+1}\,2 + \frac{(3x)^2-1}{(3x)^2+1}\,3$$

where $f'(x) = \frac{x^2-1}{x^2+1}$ by 2^{nd} FTC

$g_1'(x) = 2$
$g_2'(x) = 3$

Question 2. *(5 marks)* §5.4 #18 Find the average value of

$f(\theta) = \sec\theta\tan\theta$ on the interval $[0, \frac{\pi}{4}]$

$$\text{Avg value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{\frac{\pi}{4}-0}\int_0^{\frac{\pi}{4}} \sec\theta\tan\theta\,d\theta$$

$$= \frac{4}{\pi}\Big[\sec\theta\Big]_0^{\pi/4}$$

$$= \frac{4}{\pi}\left[\sec\frac{\pi}{4} - \sec 0\right]$$

$$= \frac{4}{\pi}\left[\sqrt{2}-1\right] = \frac{4\sqrt{2}-4}{\pi}$$