

### Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (5 marks) §5.5 #49 Evaluate the following definite integral.

$$\int_e^{e^4} \frac{1}{x\sqrt{\ln x}} dx = \int_1^4 \frac{1}{\sqrt{u}} du = \int_1^4 u^{-\frac{1}{2}} du$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$u(e) = \ln e = 1$$

$$u(e^4) = \ln e^4 = 4$$

$$= \left[ 2u^{\frac{1}{2}} \right]_1^4$$

$$= 2\sqrt{4} - 2\sqrt{1}$$

$$= 4 - 2 = 2$$

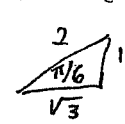
**Question 2.** (5 marks) §6.1 #20 Evaluate the following definite integral.

$$\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx = \left[ uv \right]_1^{\sqrt{3}} - \int_1^{\sqrt{3}} v du$$

$$u = \arctan\left(\frac{1}{x}\right) \quad v = x$$

$$du = \frac{1}{\left(\frac{1}{x}\right)^2 + 1} \cdot \frac{-1}{x^2} dx \quad dv = dx$$

$$du = \frac{-1}{1+x^2} dx$$

$\arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$   


$$= \left[ \arctan\left(\frac{1}{x}\right)x \right]_1^{\sqrt{3}} - \int_1^{\sqrt{3}} x \left( \frac{-1}{1+x^2} \right) dx$$

$$= \left[ \arctan\left(\frac{1}{\sqrt{3}}\right)\sqrt{3} - \arctan\left(\frac{1}{1}\right) \cdot 1 \right]$$

$$+ \int_2^4 \frac{x}{u} \frac{du}{2x}$$

$$= \frac{\pi\sqrt{3}}{6} - \frac{\pi}{4} + \frac{1}{2} \left[ \ln|u| \right]_2^4$$

$$= \frac{\pi\sqrt{3}}{6} - \frac{\pi}{4} + \frac{1}{2} \ln 4 - \frac{1}{2} \ln 2$$

$$= \frac{\pi\sqrt{3}}{6} - \frac{\pi}{4} + \ln\sqrt{2}$$

$\frac{du}{2x}$   
 $\arctan\left(\frac{1}{1}\right) = \frac{\pi}{4}$ 
