

Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §6.2 #50 Evaluate the indefinite integral.

$$\int \frac{1}{u\sqrt{5-u^2}} du = \int \frac{1}{\sqrt{5}\sin\theta \sqrt{5-(\sqrt{5}\sin\theta)^2}} \sqrt{5}\cos\theta d\theta$$

$$u = \sqrt{5}\sin\theta$$

$$du = \sqrt{5}\cos\theta d\theta$$

$$= \int \frac{\cos\theta d\theta}{\sin\theta \sqrt{5}\cos^2\theta}$$

where

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$= \int \frac{\cos\theta}{\sqrt{5}\cos\theta \sin\theta} d\theta$$

and $\theta \neq 0$

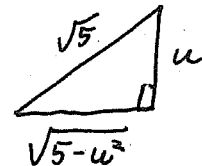
$$= \frac{1}{\sqrt{5}} \int \csc\theta d\theta$$

$$= \frac{1}{\sqrt{5}} \ln |\csc\theta + \cot\theta| + C$$

$$= \frac{1}{\sqrt{5}} \ln \left| \frac{\sqrt{5}}{u} + \frac{\sqrt{5-u^2}}{u} \right| + C$$

$$u = \sqrt{5}\sin\theta$$

$$\frac{u}{\sqrt{5}} = \sin\theta$$



Question 2. (5 marks) §6.3 #2) Evaluate the indefinite integral.

$$\int \frac{5x^2 + 3x - 2}{x^3 + 2x^2} dx \quad \frac{5x^2 + 3x - 2}{x^3 + 2x^2} = \frac{5x^2 + 3x - 2}{x^2(x+2)} = \frac{Bx + C}{x^2} + \frac{A}{(x+2)} \quad (1)$$

$$5x^2 + 3x - 2 = (Bx + C)(x+2) + Ax^2$$

Let $x = -2$

$$5(-2)^2 + 3(-2) - 2 = (B(-2) + C)(-2+2) + A(-2)^2$$

$$12 = 4A$$

$$3 = A$$

From (1)

$$\frac{x(5x^2 + 3x - 2)}{x^2(x+2)} = \frac{x(Bx + C)}{x^2} + \frac{xA}{x+2}$$

$$\lim_{x \rightarrow \infty} \left[\frac{x(5x^2 + 3x - 2)}{x^2(x+2)} \right] = \lim_{x \rightarrow \infty} \left[\frac{x(Bx + C)}{x^2} + \frac{xA}{x+2} \right]$$

$$5 = B + A$$

$$5 = B + 3$$

$$2 = B$$

Let $x = 0$

$$5(0)^2 + 3(0) - 2 = (B(0) + C)(0+2) + A(0)^2$$

$$-2 = 2C$$

$$-1 = C$$

$$\int \frac{5x^2 + 3x - 2}{x^3 + 2x^2} dx = \int \frac{2x - 1}{x^2} + \frac{3}{x+2} dx$$

$$= 2 \ln|x| + x^{-1} + 3 \ln|x+2| + C$$