

Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §6.2 # 43 Evaluate the indefinite integral.

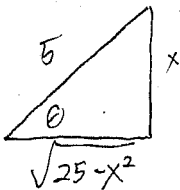
$$\int \frac{1}{x^2 \sqrt{25-x^2}} dx = \int \frac{1}{(5 \sin \theta)^2 \sqrt{25 - (5 \sin \theta)^2}} \frac{5 \cos \theta d\theta}{5 \cos \theta} = \int \frac{5 \cos \theta d\theta}{25 \sin^2 \theta \sqrt{25 \cos^2 \theta}}$$

$$x = 5 \sin \theta$$

$$dx = 5 \cos \theta d\theta$$

where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
and $\theta \neq 0$

$$\frac{\text{opp}}{\text{hyp}} = \frac{x}{5} = \sin \theta$$



$$= \int \frac{5 \cos \theta}{25 \sin^2 \theta \cdot 5 \cos \theta} d\theta$$

$$= \frac{1}{25} \int \frac{1}{\sin^2 \theta} d\theta$$

$$= \frac{1}{25} \int \csc^2 \theta d\theta$$

$$= -\frac{1}{25} \cot \theta + C$$

$$= -\frac{1}{25} \frac{\sqrt{25-x^2}}{x} + C$$

Question 2. (5 marks) §6.3 #22 Evaluate the indefinite integral.

$$\int \frac{x^2 - x + 6}{x^3 + 3x} dx$$

$$\textcircled{1} \frac{x^2 - x + 6}{x(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}$$

$$x^2 - x + 6 = A(x^2 + 3) + (Bx + C)x$$

Let $x = 0$

$$6 = A(0^2 + 3) + (B(0) + C)0$$

$$2 = A$$

From $\textcircled{1}$

$$\frac{x(x^2 - x + 6)}{x(x^2 + 3)} = \frac{xA}{x} + \frac{x(Bx + C)}{x^2 + 3}$$

$$\lim_{x \rightarrow \infty} \left[\frac{x(x^2 - x + 6)}{x(x^2 + 3)} \right] = \lim_{x \rightarrow \infty} \left[\frac{xA}{x} + \frac{x(Bx + C)}{x^2 + 3} \right]$$

$$1 = A + B$$

$$1 = 2 + B$$

$$-1 = B$$

Let $x = -1$

$$(-1)^2 - (-1) + 6 = 2((-1)^2 + 3) + (B(-1) + C)(-1)$$

$$8 = 8 + (1 + C)$$

$$-1 = C$$

$$\therefore \int \frac{x^2 - x + 6}{x^3 + 3x} dx = \int \frac{2}{x} + \frac{-x - 1}{x^2 + 3} dx$$

$$= 2 \ln|x| - \frac{1}{2} \ln(x^2 + 3) - \frac{1}{\sqrt{3}} \arctan \frac{x}{\sqrt{3}} + C$$