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## Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (5 marks) §6.2 #31 Evaluate the indefinite integral.

$$\begin{aligned} \int \cot^3 \alpha \csc^3 \alpha \, d\alpha &= \int \cot^2 \alpha \csc^2 \alpha \cot \alpha \csc \alpha \, d\alpha \\ &= \int (\csc^2 \alpha - 1) \csc^2 \alpha \cot \alpha \csc \alpha \, d\alpha \\ &= \int (u^2 - 1) u^2 \, du \\ &= \int u^4 - u^2 \, du \\ &= -\frac{u^5}{5} + \frac{u^3}{3} + C \\ &= -\frac{\csc^5 \alpha}{5} + \frac{\csc^3 \alpha}{3} + C \end{aligned}$$

$$\begin{aligned} u &= \csc \alpha \\ du &= -\cot \alpha \csc \alpha \, d\alpha \end{aligned}$$

**Question 2.** (5 marks) §6.5 #8 Use the Trapezoidal Rule to approximate the given integral with  $n = 4$ ,

$$\int_0^{1/2} \sin x^2 \, dx. \quad \Delta x = \frac{b-a}{n} = \frac{1/2 - 0}{4} = \frac{1}{8} \quad \begin{aligned} x_i &= a + i\Delta x \\ x_i &= \frac{i}{8} \end{aligned} \quad \begin{aligned} x_0 &= 0, x_1 = \frac{1}{8}, x_2 = \frac{2}{8}, x_3 = \frac{3}{8}, x_4 = \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} &\approx \frac{\Delta x}{2} \left[ f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4) \right] \\ &= \frac{(1/8)}{2} \left[ f(0) + 2f(1/8) + 2f(2/8) + 2f(3/8) + f(4/8) \right] \\ &= \frac{1}{16} \left[ \sin 0 + 2 \sin\left(\frac{1}{64}\right) + 2 \sin\left(\frac{1}{16}\right) + 2 \sin\left(\frac{9}{64}\right) + \sin\left(\frac{1}{4}\right) \right] \\ &\doteq 0.04274345426 \end{aligned}$$