

Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §6.2 #32 Evaluate the indefinite integral.

$$\begin{aligned}
 \int \csc^4 x \cot^6 x \, dx &= \int \cot^6 x \csc^2 x \csc^2 x \, dx && u = \cot x \\
 &= \int \cot^6 x (1 + \cot^2 x) \csc^2 x \, dx && du = -\csc^2 x \, dx \\
 &= -\int u^6 (1 + u^2) \, du && -du = \csc^2 x \, dx \\
 &= -\int u^6 + u^8 \, du \\
 &= -\left[\frac{u^7}{7} + \frac{u^9}{9} \right] + C \\
 &= -\frac{\cot^7 x}{7} - \frac{\cot^9 x}{9} + C
 \end{aligned}$$

Question 2. (5 marks) §6.5 #8 Use Simpson's Rule to approximate the given integral with $n = 4$,

$$\int_0^{1/2} \sin x^2 \, dx. \quad \Delta x = \frac{b-a}{n} = \frac{1/2 - 0}{4} = \frac{1}{8} \quad x_i = a + i\Delta x \quad x_0 = 0, x_1 = \frac{1}{8}, x_2 = \frac{2}{8}, x_3 = \frac{3}{8}, x_4 = \frac{4}{8} = \frac{1}{2}$$

$$\begin{aligned}
 &\approx \frac{\Delta x}{3} \left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4) \right] \\
 &= \frac{(1/8)}{3} \left[f(0) + 4f(1/8) + 2f(2/8) + 4f(3/8) + f(4/8) \right] \\
 &= \frac{1}{24} \left[\sin 0 + 4\sin\left(\frac{1}{64}\right) + 2\sin\left(\frac{1}{16}\right) + 4\sin\left(\frac{9}{64}\right) + \sin\left(\frac{1}{4}\right) \right] \\
 &= 0.04147783088
 \end{aligned}$$