Dawson College: **Calculus II**: 201-NYB-05-S2: Winter 2010

Name: Yann Lamontagne
Student ID: ____________

Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §3.7 #34 Find the limit.

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}$$

let $y = \lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}$

$$\ln y = \ln\left(\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}\right)$$

$$\ln y = \lim_{x\to\infty} \ln\left(1+\frac{a}{x}\right)^{bx}$$

$$\ln y = \lim_{x\to\infty} bx\ln\left(1+\frac{a}{x}\right) \quad \text{I.F.} \quad \infty\cdot 0$$

$$\ln y = \lim_{x\to\infty} b\,\frac{\ln\left(1+\frac{a}{x}\right)}{\frac{1}{bx}} \quad \text{I.F.} \quad \frac{0}{0}$$

$$\ln y = \lim_{x\to\infty} b\,\frac{\frac{1}{1+a/x}\,\frac{-a}{\cancel{x^2}}}{\cancel{\frac{-1}{x^2}}} \quad \text{by } \hat{H}$$

$$\ln y = \lim_{x\to\infty} ab\,\cancelto{1}{\frac{1}{1+a/x}}$$

$$\ln y = ab$$

$$y = e^{ab}$$

Question 2. *(5 marks)* §6.6 #32 Determine if the integral is convergent or divergent. Evaluate those that are convergent.

$$\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$$

Infinite discontinuity at $x=0$.

$u = \ln x \quad du = \frac{1}{x}dx$

$v = 2\sqrt{x} \quad dv = \frac{1}{\sqrt{x}}dx$

$$= \lim_{a\to 0^+}\int_a^1 \frac{\ln x}{\sqrt{x}}$$

$$= \lim_{a\to 0^+}\left[\left[uv\right]_a^1 - \int_a^1 v\,du\right]$$

$$= \lim_{a\to 0^+}\left[\left[2\sqrt{x}\ln x\right]_a^1 - \int_a^1 \frac{2\sqrt{\cancel{x}}}{x^{1/2}}dx\right]$$

$$= \lim_{a\to 0^+}\left[\underset{\substack{\shortparallel\\ 0}}{2\sqrt{1}\ln 1} - 2\sqrt{a}\ln a - \left[4\sqrt{x}\right]_a^1\right]$$

$$= \lim_{a\to 0^+}\left[2\sqrt{a}\ln a - 4 + \cancelto{0}{4\sqrt{a}}\right]$$

$$= \lim_{a\to 0^+} 2\sqrt{a}\ln a - 4 \quad \text{I.F.} \quad 0\cdot-\infty$$

$$= -4 + \lim_{a\to 0^+}\frac{2\ln a}{\frac{1}{\sqrt{a}}} \quad \text{I.F} \quad \frac{-\infty}{\infty}$$

$$= -4 + \lim_{a\to 0^+}\frac{2\frac{1}{a}}{\frac{-1}{2}a^{-3/2}} \quad \text{by } \hat{H}$$

$$= -4 + \lim_{a\to 0^+} -4\,\cancelto{0}{\frac{a^{3/2}}{a}}$$

$$= -4$$