

Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §3.7 #32 Find the limit.

$$\lim_{x \rightarrow \infty} x^{(\ln 2)/(1+\ln x)} \quad \text{Let } y = \lim_{x \rightarrow \infty} x^{(\ln 2)/(1+\ln x)}$$

$$\ln y = \ln \left(\lim_{x \rightarrow \infty} x^{(\ln 2)/(1+\ln x)} \right)$$

$$\ln y = \lim_{x \rightarrow \infty} \ln x^{(\ln 2)/(1+\ln x)} \quad \text{since } \ln \text{ is continuous}$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{\ln 2}{1+\ln x} \ln x \quad \text{l.f. } \frac{\infty}{\infty}$$

$$\ln y = \lim_{x \rightarrow \infty} \frac{(\ln 2)(\frac{1}{x})}{\frac{1}{x}} \quad \text{by } \hat{H}$$

$$\ln y = \ln 2$$

$$y = 2$$

Question 2. (5 marks) §6.6 #18 Determine if the integral is convergent or divergent. Evaluate those that are convergent.

$$\int_{-\infty}^6 re^{r/3} dr = \lim_{a \rightarrow -\infty} \int_a^6 re^{r/3} dr \quad \begin{matrix} u=r & du=dr \\ v=3e^{r/3} & dv=e^{r/3} dr \end{matrix}$$

$$= \lim_{a \rightarrow -\infty} \left[\left[uv \right]_a^6 - \int_a^6 v du \right]$$

$$= \lim_{a \rightarrow -\infty} \left[\left[3re^{r/3} \right]_a^6 - \int_a^6 3e^{r/3} dr \right]$$

$$= \lim_{a \rightarrow -\infty} \left[3(6)e^{6/3} - 3ae^{a/3} - \left[9e^{r/3} \right]_a^6 \right]$$

$$= 18e^2 - \lim_{a \rightarrow -\infty} \left[3ae^{a/3} - 9e^{6/3} + 9e^a \right]$$

$$= 9e^2 - \lim_{a \rightarrow -\infty} 3ae^{a/3} \quad \text{l.f. } -\infty \cdot 0$$

$$= 9e^2 - \lim_{a \rightarrow -\infty} \frac{3a}{e^{-a/3}} \quad \text{l.f. } \frac{0}{0}$$

$$= 9e^2 - \lim_{a \rightarrow -\infty} \frac{3}{\frac{1}{3}e^{-a/3}} \quad \text{by } \hat{H}$$

$$= 9e^2$$

\therefore converges to $9e^2$