Dawson College: **Calculus II**: 201-NYB-05-S02: Winter 2010

Name: _____
Student ID: _____

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulae:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(5 marks)* Evaluate using the definition of the definite integral

$$\int_1^2 -3x^2 + 2x - 1 \, dx.$$

Question 2. *(5 marks)* Evaluate the definite integral:

$$\int_1^2 \frac{(x+1)(x-2)}{\sqrt{x}} - e^x \, dx$$

Question 3. *(5 marks)* Evaluate the indefinite integral:

$$\int z^2 \sqrt{z-1} \, dz$$

Question 4. *(5 marks)* Find the average value of the function

$$f(x) = \frac{x}{x^4 + 9}$$

on the interval $[0, \sqrt{3}]$

Question 5. *(5 marks + 1 bonus mark to simplify completely)* Evaluate the expression:

$$\frac{d}{dx}\left[\int_{-\sin x}^{\sin x} t\cos t^9 \, dt\right]$$

Question 6. *(5 marks)* Suppose $f(x)$ is continous over the real numbers and

$$\int_0^1 f(x)\,dx = 131.$$

Evaluate

$$\int_0^{\pi/6} \cos(3x)\, f(\sin(3x))\,dx.$$

Question 7. *(5 marks)* Evaluate the indefinite integral:

$$\int x^2 \arcsin x\,dx$$

Question 8. *(5 marks)* Evaluate the definite integral:

$$\int_{\pi/6}^{\pi/8} \theta \sec 2\theta \tan 2\theta \; d\theta$$

Question 9. *(5 marks)* Prove: If $f(x)$ is a continous function then

$$\int_a^b f(x) + g(x)\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx.$$

Bonus Question. *(3 marks)*
If $f(x)$ is a continuous function on a certain domain and satisfies

$$0 = \int_{101}^{x} f(t)\,dt - \operatorname{arcsec}(\ln x) - \int_{x}^{102} (t^2+1) f(t)\,dt$$

then find $f(x)$ and state its domain.