

Test 2

This test is graded out of 53 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (3 marks) Evaluate the definite integral:

$$\int_{\pi/4}^{\pi/3} \sin^2 z \, dz$$

Question 2. (5 marks) Evaluate the improper integral or show it diverges:

$$\int_5^{\infty} \frac{1}{(x-2)\sqrt{x^2-4x-5}} \, dx$$

Question 3. (5 marks) Evaluate the indefinite integral:

$$\int \csc^4 2\theta \cot^2 2\theta \, d\theta$$

Question 4. (5 marks) Evaluate the indefinite integral:

$$\int \frac{\sqrt{4x^2 - 9}}{x^4} \, dx$$

Question 5. (5 marks) Sketch and find the total area of the region(s) bounded by the graphs of $y = \frac{20}{x^2+4}$, $y = 5 - x$.

Question 6. (5 marks) Evaluate the limit:

$$\lim_{x \rightarrow 2^-} (\ln(3-x))^{2-x}$$

Question 7. (5 marks) Evaluate the indefinite integral:

$$\int \frac{1}{x^3 - 9x} dx$$

Question 8. Only answer one of the following two questions.

a. (5 marks + 1 bonus mark for complete and correct solution) Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\int_e^{e^{-x}} \ln t \, dt + \int_0^{\sin x} \sec t \, dt}{\cos ax - \cos bx}$$

where $a, b \neq 0$ and $a \neq b$.

b. (5 marks) Find the value(s) of a and b such that

$$\lim_{x \rightarrow 0} \left(\frac{\tan 2x}{x^2} + a + \frac{b}{x} \right) = 0$$

Question 9. Only answer one of the following two questions.

- a. (5 marks) Find the value(s) of c such that the area of the region bounded by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 72. Sketch the graphs of the functions.
- b. (5 marks) Find the value of a such that the line $x = a$ bisects the area under the curve $y = 1/x^3$ on $1 \leq x < \infty$. Sketch the graphs of the function and relation.

Question 10. Only answer one of the following two questions.

a. (5 marks + 2 bonus marks for complete and correct solution) If $f(x)$ is a quartic function such that $f(0) = 2$, $f''(0) = 12$ and

$$\int \frac{f(x)}{x^2(x+1)^2(x^2+1)} dx$$

is a rational function, find the value of $f^{(4)}(x)$.

b. (5 marks) Find the value of the constant C for which the integral

$$\int_0^{\infty} \left(\frac{x}{x^2+1} - \frac{C}{2x+1} \right) dx$$

converges. Evaluate the integral for this value of C .

Bonus Question. Only answer one of the following two bonus questions.

a. (5 marks) Show that if $f(x)$ is a polynomial of degree 3 or lower, then Simpson's Rule gives the exact value of

$$\int_a^b f(x) dx$$

b. (2 marks) Show that

$$\frac{1}{3}T_n + \frac{2}{3}M_n = S_{2n}$$