Dawson College: **Calculus II**: 201-NYB-05-S02: Winter 2010

Name: ____________________

Student ID: ____________________

Test 3

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of $y = \sqrt{x+1}+1$, $y = 6-x$ and $y = 1$ rotated about the line $y = 6$.

Question 2. *(5 marks)* A tank which has the shape of a cone with its tip upwards has a $10m$ diameter on the bottom and a height of $6m$. Set up the integral to find the work required to empty the tank of with a liquid with an arbitrary density ρ from a pump $3m$ above the tip of the tank.

Question 3. *(5 marks)* Find the arc length of the graph of

$$y = \frac{x^2}{2} - \frac{\ln x}{4}$$

on the interval $[1, e]$

Question 4. *(5 marks)* Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{n!\sin\left(\frac{1}{n}\right)}{(n-1)!}$$

Question 5. *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{2+\pi^{n+1}}{e^{n-1}}$$

Question 6. *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=2}^{\infty} [\operatorname{arcsec}(n-1) - \operatorname{arcsec}(n+1)]$$

Question 7. *(5 marks)* If the n^{th} partial sum of a series $\sum_{n=1}^{\infty} a_n$ is

$$S_n = \arctan n$$

find a_n and $\sum_{n=1}^{\infty} a_n$.

Question 8. *(5 marks)* Determine whether the following series converges or diverges. Justify your answer.

$$\sum_{n=3}^{\infty} \frac{\sqrt{n+1}}{n\sqrt{n-\sqrt{n}-1}}$$

Question 9. *(5 marks)* Determine whether the following series converges or diverges. Justify your answer.

$$\sum_{n=5}^{\infty} \frac{n(2n)!}{3^n n!}$$

Question 10. *(5 marks)* Prove: If $p > 1$ then

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

converges.

Bonus Question.

a. *(1 mark)* State the $K(\varepsilon)$ definition of the limit of a sequence.

b. *(3 marks)* Using the $K(\varepsilon)$ definition show that

$$\lim_{n\to\infty} \frac{(-1)^n n}{n^2+1} = 0$$

c. *(3 marks)* Using the $K(\varepsilon)$ definition show: If a_n converges to a and b_n converges to b then $a_n + b_n$ converges to $a+b$.