Final Examination (201-NYC-05) Winter 2005 (Regular)

3

a) $\|\vec{u}\times\vec{v}\|^2 = \left\| \left( \begin{vmatrix} -1 & 3 \\ 3 & 5 \end{vmatrix}, -\begin{vmatrix} 2 & -2 \\ 3 & 5 \end{vmatrix}, \begin{vmatrix} 2 & -2 \\ -1 & 3 \end{vmatrix} \right) \right\|^2 = \|(-14, -16, 4)\|^2$

$\begin{matrix} 2 & -2 \\ -1 & 3 \\ 3 & 5 \end{matrix}$

$= \left(\sqrt{(-14)^2 + (-16)^2 + 4^2}\right)^2$

$= 468$

b) $(2\vec{u})\cdot(3\vec{w} - 4\vec{v}) = (4,-2,6)\cdot\big((-3,6,12) - (-8,12,20)\big)$

$= (4,-2,6)\cdot(5,-6,-8)$

$= 20 + 12 - 48$

$= -16$

c) $\vec{u}\cdot\vec{v} = \|\vec{u}\|\|\vec{v}\|\cos\theta$

$(2,-1,3)\cdot(-2,3,5) = \|(2,-1,3)\|\,\|(-2,3,5)\|\cos\theta$

$-4 - 3 + 15 = \sqrt{4+1+9}\,\sqrt{4+9+25}\cos\theta$

$8 = \sqrt{14}\,\sqrt{38}\cos\theta$

$\theta = \cos^{-1}\left(\frac{8}{\sqrt{14}\sqrt{38}}\right)$

$\theta \doteq 70^\circ$

d) $\mathrm{proj}_{\vec{v}}(2\vec{w}) = \frac{2\vec{w}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{v}$

$= 2\,\frac{(-1,2,4)\cdot(-2,3,5)}{(-2,3,5)\cdot(-2,3,5)}(-2,3,5)$

$= 2\,\frac{(2+6+20)}{4+9+25}(-2,3,5)$

$= \frac{56}{38}(-2,3,5) = \frac{28}{19}(-2,3,5)$

e) $V = |\vec{u}\cdot(\vec{v}\times\vec{w})| = \left| \begin{vmatrix} 2 & -1 & 3 \\ -2 & 3 & 5 \\ -1 & 2 & 4 \end{vmatrix} \right| = \left| 2(-1)^{1+1}\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} + (-1)(-1)^{1+2}\begin{vmatrix} -2 & 5 \\ -1 & 4 \end{vmatrix} + 3(-1)^{1+3}\begin{vmatrix} -2 & 3 \\ -1 & 2 \end{vmatrix} \right|$

$= \left| 2\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} + \begin{vmatrix} -2 & 5 \\ -1 & 4 \end{vmatrix} + 3\begin{vmatrix} -2 & 3 \\ -1 & 2 \end{vmatrix} \right|$

$= |2[12-10] + [-8+5] + 3[-4+3]|$

$= |4 - 3 - 3|$

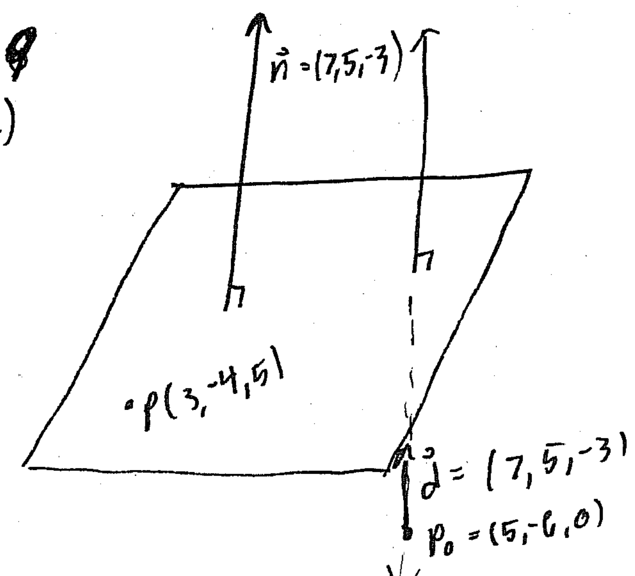
$= |-2|$

$= 2$

f) $\frac{\vec{u}}{\|\vec{u}\|} = \frac{(2,-1,3)}{\|(2,-1,3)\|} = \frac{(2,-1,3)}{\sqrt{4+1+9}} = \frac{(2,-1,3)}{\sqrt{14}}$

Q8

a)




$7x + 5y - 3z = d$

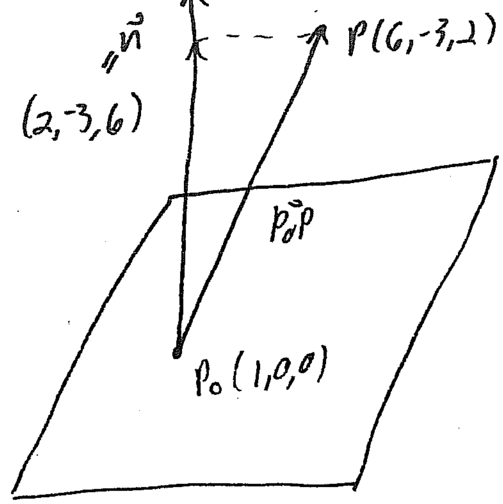
$7(3) + 5(-4) - 3(5) = d$

$21 - 20 - 15 = d$

$-14 = d$

$\therefore 7x + 5y - 3z = -14$

b)




To find $P_0$ let $y=z=0$

$$2x-3(0)+6(0)=-2$$
$$x=-1$$

$$\vec{P_0P} = P - P_0 = (-1,0,0) - (6,-3,2)$$
$$= (-7, 3, -2)$$

$$\vec{d} = \|\text{proj}_{\vec{n}} \vec{P_0P}\| = \left\| \frac{(-7,3,-2)\cdot(2,-3,6)}{(2,-3,6)\cdot(2,-3,6)} (2,-3,6) \right\|$$

$$= \left\| \frac{-14-9-12}{4+9+36} (2,-3,6) \right\|$$

$$= \left\| \frac{35}{49} (2,-3,6) \right\|$$

$$= \sqrt{\frac{(70)^2+(105)^2+(210)^2}{49^2}} = 5$$

Q10)

$$\left.\begin{aligned} -3+t &= 1-3s \\ -7+3t &= 3-8s \\ 1-2t &= -1+3s \end{aligned}\right\} \iff \left\{\begin{aligned} t+3s &= 4 \\ 3t+8s &= 10 \\ -2t-3s &= -2 \end{aligned}\right. \qquad \begin{bmatrix} 1 & 3 & 4 \\ 3 & 8 & 10 \\ -2 & -3 & -2 \end{bmatrix}$$

$$\sim \begin{matrix} -3R_1+R_2 \to R_2 \\ 2R_1+R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & 3 & 4 \\ 0 & -1 & -2 \\ 0 & 3 & 6 \end{bmatrix} \sim \quad 3R_2+R_3 \to R_3 \begin{bmatrix} 1 & 3 & 4 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} 3R_2+R_1 \to R_1 \\ \sim -R_2 \end{matrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} t &= -2 \\ s &= 2 \end{aligned}$$

$$\therefore \begin{aligned} x &= -3-2 = -5 \\ y &= -7+3(-2) = -13 \\ z &= 1-2(-2) = 5 \end{aligned} \qquad \therefore (-5,-13,5).$$

Q11

$$\begin{bmatrix} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \end{bmatrix} \sim \underset{-2R_1+R_2 \to R_2}{} \begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 5 & -10 & -5 \end{bmatrix}$$

$$\sim \underset{\frac{1}{5}R_2}{} \begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 1 & -2 & -1 \end{bmatrix}$$

$$\sim \underset{2R_2+R_1 \to R_1}{} \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -2 & -1 \end{bmatrix}$$

$\therefore \quad z = t$

$$\left.\begin{aligned} x - t &= 2 \\ y - 2t &= -1 \\ z &= t \end{aligned}\right\} \quad \begin{aligned} x &= 2 + t \\ y &= -1 + 2t \\ z &= t \end{aligned}$$

$\therefore \quad (x, y, z) = (2, -1, 0) + t(1, 2, 1).$