Dawson College: Linear Algebra:

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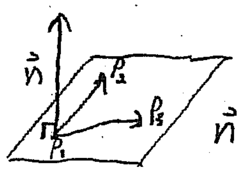
Assignment 1

This assignment is graded out of 9 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Find the equation of the plane passing throught the points: $P_1(1,2,3)$, $P_2(-2,0,-1)$, $P_3(0,-1,0)$.



$\vec{P_1P_2} = P_2 - P_1 = (-2,0,-1)-(1,2,3) = (-3,-2,-4)$

$\vec{P_1P_3} = P_3 - P_1 = (0,-1,0)-(1,2,3) = (-1,-3,-3)$

$\vec{n} = \vec{P_1P_2} \times \vec{P_1P_3} = \left(\begin{vmatrix} -2 & -3 \\ -4 & -3 \end{vmatrix}, - \begin{vmatrix} -3 & -1 \\ -4 & -3 \end{vmatrix}, \begin{vmatrix} -3 & -1 \\ -2 & -3 \end{vmatrix} \right) = (-6, -5, +7)$

$\begin{matrix} -3 & -1 \\ -2 & -3 \\ -4 & -3 \end{matrix}$

$-6x - 5y + 7z = d$

$-6(0) - 5(-1) + 7(0) = d$

$5 = d$

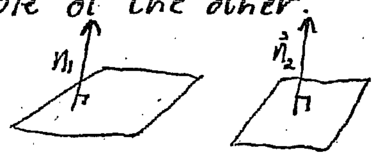
$\therefore -6x - 5y + 7z = 5$

Question 2. Determine if the two planes are parallel: $2x+y-z=101$ and $x+2y-z=-101$.

2 planes are $\parallel$ iff their normals are a multiple of the other.

i.e. $\vec{n}_1 = (2,1,-1)$, $\vec{n}_2 = (1,2,-1)$ $\quad \vec{n}_1 \neq k\vec{n}_2$

$\therefore$ not parallel

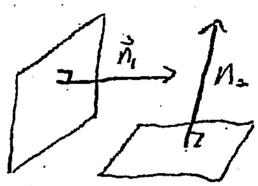


Question 3. Determine if the two planes are perpendicular: $2x-z=101$ and $2y=-101$.

2 planes are $\perp$ iff their normals are perpendicular

i.e. $\vec{n}_1 = (2,0,-1)$, $\vec{n}_2 = (0,2,0)$ $\quad n_1 \cdot n_2 = 0$

$\therefore$ two planes are perpendicular

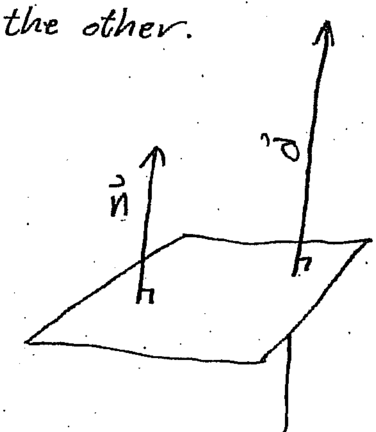


Question 4. Determine if the line and the plane are perpendicular: $(x,y,z)=(1,2,2)+t(2,-1,2)$ and $-4x+2y-4z=103$.

a line and a plane are perpendicular if the normal and the direction vector are a multiple of the other.

i.e. $\vec{n} = (-4,2,-4)$ $\quad \vec{d} = (2,-1,2)$ $\quad \vec{n} = -2\vec{d}$

$\therefore$ the line and the plane are perpendicular

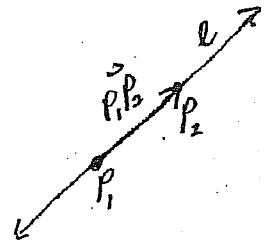


Question 5 Find the equation of the line passing through the given points: $P_1(8,-3,4)$ and $P_2(2,1,2)$.

$\ell = P_1 + t\vec{P_1P_2}$ where $\vec{P_1P_2} = P_2 - P_1$

$= (2,1,2) - (8,-3,4)$

$= (-6,4,-2)$



Question 6 Find the equation for the line of intersection of the given planes: $-x+2y-z=3$ and $2x-y+3z=4$.

Find the line of intersection by solving the system.

$$\begin{bmatrix} -1 & 2 & -1 & 3 \\ 2 & -1 & 3 & 4 \end{bmatrix}$$

$$\sim \begin{array}{c} \\ 2R_1+R_2\to R_2 \end{array} \begin{bmatrix} -1 & 2 & -1 & 3 \\ 0 & 3 & 1 & 10 \end{bmatrix}$$

$$\sim \begin{array}{c} 3R_1 \\ \\ \end{array} \begin{bmatrix} -3 & 6 & -3 & 9 \\ 0 & 3 & 1 & 10 \end{bmatrix}$$

$$\sim \begin{array}{c} -2R_2+R_1\to R_1 \\ \\ \end{array} \begin{bmatrix} -3 & 0 & -5 & -11 \\ 0 & 3 & 1 & 10 \end{bmatrix}$$

$$\begin{array}{c} -\frac{1}{3}R_1 \\ \frac{1}{3}R_2 \end{array} \begin{bmatrix} 1 & 0 & 5/3 & 11/3 \\ 0 & 1 & 1/3 & 10/3 \end{bmatrix}$$

$z=t$ since z is the free variable

$x = \frac{11}{3} - \frac{5}{3}t$

$y = \frac{10}{3} - \frac{1}{3}t$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{11}{3} \\ \frac{10}{3} \\ 0 \end{pmatrix} + t \begin{pmatrix} -5/3 \\ -1/3 \\ 1 \end{pmatrix}$

Question 7. Find the equation of the plane that passes throught the point $(2,-3,4)$ and is parallel to the plane $2x-y-z-301=0$

Two planes are parallel iff their normal are a multiple of each other.

$\therefore$ $2x-y-z=d$ sub $(2,-3,4)$ $2(2)-(-3)-4=d$

$3=d$

$\therefore 2x-y-z=3$

Question 8. Find an equation for the plane through $(3,-4,3)$ that is perpendicular to the line of intersection of the planes $2x-y+5z=1$ and $-3x+y+4z=3$.

Find the line of intersection

$$\begin{bmatrix} 2 & -1 & 5 & 1 \\ -3 & 1 & 4 & 3 \end{bmatrix}$$

$$\sim \begin{array}{c} \\ 2R_2 \end{array} \begin{bmatrix} 2 & -1 & 5 & 1 \\ -6 & 2 & 8 & 6 \end{bmatrix}$$

$$\sim \begin{array}{c} \\ 3R_1+R_2\to R_2 \end{array} \begin{bmatrix} 2 & -1 & 5 & 1 \\ 0 & -1 & 23 & 9 \end{bmatrix}$$

$$\sim \begin{array}{c} \\ -R_2 \end{array} \begin{bmatrix} 2 & -1 & 5 & 1 \\ 0 & 1 & -23 & -9 \end{bmatrix}$$

$$\sim \begin{array}{c} R_1+R_2\to R_1 \\ \\ \end{array} \begin{bmatrix} 2 & 0 & -18 & -8 \\ 0 & 1 & -23 & -9 \end{bmatrix}$$

$$\sim \begin{array}{c} \frac{1}{2}R_1 \\ \\ \end{array} \begin{bmatrix} 1 & 0 & -9 & -4 \\ 0 & 1 & -23 & -9 \end{bmatrix}$$

$z=t$ since z is a free variable

$x=-4+9t$

$y=-9+23t$

$\therefore \vec{d} = (9, 23, 1)$ which is normal of the plane

$9x+23y+z=d$

$9(3)+23(-4)+3=d$

$-62=d$

$\therefore +14x+23y+z=-62$

Question 9. Find the intersection of the line $(x,y,z) = (1,2,1)+t(-2,0,1)$ and the plane $x-y+3z=6$.

$\ell = \begin{cases} x = 1-2t \\ y = 2 \\ z = 1+t \end{cases}$ sub into eqn.

$(1-2t)-(2)+3(1+t)=6$

$1-2t-2+3+3t = 6$

$t = 4$

$\therefore$ intersection at $t=4$

$x = 1-2(4)$ $x=-7$

$y=2$ $y=2$

$z=1+4$ $z=5$

$\therefore$ intersection at $(-7,2,5)$