

Assignment 2

This assignment is graded out of 10 marks. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Maximize $p = 8x_1 + 9x_2 + 4x_3$ subject to the constraints

$$\begin{array}{lcl}
 x_1 + x_2 + 2x_3 \leq 2 & \text{into} & x_1 + x_2 + 2x_3 + s_1 = 2 \\
 2x_1 + 3x_2 + 4x_3 \leq 3 & \Rightarrow & 2x_1 + 3x_2 + 4x_3 + s_2 = 3 \\
 7x_1 + 6x_2 + 2x_3 \leq 8 & \text{eqn.} & 7x_1 + 6x_2 + 2x_3 + s_3 = 8 \\
 & & -8x_1 - 9x_2 - 4x_3 + p = 0
 \end{array}$$

Simplex matrix:

$$\left[\begin{array}{cccc|cccc}
 1 & 1 & 2 & 1 & 0 & 0 & 0 & 2 \\
 2 & 3 & 4 & 0 & 1 & 0 & 0 & 3 \\
 7 & 6 & 2 & 0 & 0 & 1 & 0 & 8 \\
 -8 & -9 & -4 & 0 & 0 & 0 & 1 & 0
 \end{array} \right]
 \begin{array}{l}
 r = 2/1 = 2 \\
 r = 3/3 = 1 \\
 r = 8/6 = 1.\bar{3}
 \end{array}$$

pivot row ← pivot column

$$\frac{1}{3}R_2 \left[\begin{array}{cccc|cccc}
 1 & 1 & 2 & 1 & 0 & 0 & 0 & 2 \\
 2/3 & 1 & 4/3 & 0 & 1/3 & 0 & 0 & 1 \\
 7 & 6 & 2 & 0 & 0 & 1 & 0 & 8 \\
 -8 & -9 & -4 & 0 & 0 & 0 & 1 & 0
 \end{array} \right]$$

$$\begin{array}{l}
 -R_2 + R_1 \rightarrow R_1 \\
 -R_2 + R_3 \rightarrow R_3 \\
 +9R_2 + R_4 \rightarrow R_4
 \end{array}
 \left[\begin{array}{cccc|cccc}
 1/3 & 0 & 2/3 & 1 & -1/3 & 0 & 0 & 1 \\
 2/3 & 1 & 4/3 & 0 & 1/3 & 0 & 0 & 1 \\
 3 & 0 & -6 & 0 & -2 & 1 & 0 & 2 \\
 -2 & 0 & 8 & 0 & 3 & 0 & 1 & 9
 \end{array} \right]
 \begin{array}{l}
 r = 1/1/3 = 3 \\
 r = 1/2/3 = 3/2 \\
 r = 3/3 \leftarrow \text{pivot row}
 \end{array}$$

pivot column

$$\frac{1}{3}R_3 \left[\begin{array}{cccc|cccc}
 1/3 & 0 & 2/3 & 1 & -1/3 & 0 & 0 & 1 \\
 2/3 & 1 & 4/3 & 0 & 1/3 & 0 & 0 & 1 \\
 1 & 0 & -2 & 0 & -2/3 & 1/3 & 0 & 2/3 \\
 -2 & 0 & 8 & 0 & 3 & 0 & 1 & 9
 \end{array} \right]$$

∴ max is $31/3$
 at $x_1 = 2/3, x_2 = 5/9, x_3 = 0$

$$\begin{array}{l}
 -\frac{1}{3}R_3 + R_1 \rightarrow R_1 \\
 -\frac{7}{3}R_3 + R_2 \rightarrow R_2 \\
 2R_3 + R_4 \rightarrow R_4
 \end{array}
 \left[\begin{array}{cccc|cccc}
 0 & 0 & 4/3 & 1 & -1/9 & -1/9 & 0 & 7/9 \\
 0 & 1 & 8/3 & 0 & 7/9 & -2/9 & 0 & 5/9 \\
 1 & 0 & -2 & 0 & -2/3 & 1/3 & 0 & 2/3 \\
 0 & 0 & 4 & 0 & 5/3 & 2/3 & 1 & 31/3
 \end{array} \right]$$

Question 2. (5 marks) Maximize $p = x_1 + 2x_2 + 4x_3 + 5x_4$ subject to the constraints

$$\begin{aligned} x_1 + x_2 + x_4 &\leq 44 && \text{into} && x_1 + x_2 & + x_4 + S_1 & = 44 \\ 2x_1 + x_2 + 2x_3 + 5x_4 &\leq 200 && \Rightarrow && 2x_1 + x_2 + 2x_3 + 5x_4 & + S_2 & = 200 \\ x_1 + x_3 &\leq 50 && \text{eqn} && x_1 & + x_3 & + S_3 & = 50 \\ &&& && -x_1 - 2x_2 - 4x_3 - 5x_4 & & + p & = 0 \end{aligned}$$

$$\left[\begin{array}{cccc|cccc|c} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 44 \\ 2 & 1 & 2 & 5 & 0 & 1 & 0 & 0 & 200 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ -1 & -2 & -4 & -5 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \quad \begin{aligned} r &= 44/1 = 44 \\ r &= 200/5 = 40 \leftarrow \text{pivot row} \end{aligned}$$

↑ pivot column

$$\frac{1}{5}R_2 \rightarrow R_2 \left[\begin{array}{cccc|cccc|c} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 44 \\ 2/5 & 1/5 & 2/5 & 1 & 0 & 1/5 & 0 & 0 & 40 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ -1 & -2 & -4 & -5 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$\begin{aligned} -R_2 + R_1 \rightarrow R_1 & \left[\begin{array}{cccc|cccc|c} 3/5 & 4/5 & -2/5 & 0 & 1 & -1/5 & 0 & 0 & 4 \\ 2/5 & 1/5 & 2/5 & 1 & 0 & 1/5 & 0 & 0 & 40 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ 1 & -1 & -2 & 0 & 0 & 1 & 0 & 1 & 200 \end{array} \right] \quad \begin{aligned} r &= 40/2/5 = 100 \\ r &= 50/1 = 50 \leftarrow \text{pivot row} \end{aligned} \\ 5R_2 + R_4 \rightarrow R_4 & \left[\begin{array}{cccc|cccc|c} 1 & -1 & -2 & 0 & 0 & 1 & 0 & 1 & 200 \end{array} \right] \end{aligned}$$

↑ pivot column

$$\begin{aligned} \frac{2}{5}R_3 + R_1 \rightarrow R_1 & \left[\begin{array}{cccc|cccc|c} 1 & 4/5 & 0 & 0 & 1 & -1/5 & 2/5 & 0 & 24 \\ 0 & 1/5 & 0 & 1 & 0 & 1/5 & -1/5 & 0 & 20 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ 3 & -1 & 0 & 0 & 0 & 1 & 2 & 1 & 300 \end{array} \right] \quad \begin{aligned} r &= 24/4/5 = 30 \leftarrow \text{pivot row} \\ r &= 20/1/5 = 100 \end{aligned} \\ -\frac{2}{5}R_3 + R_2 \rightarrow R_2 & \\ 2R_3 + R_4 \rightarrow R_4 & \end{aligned}$$

↑ pivot column

$$\frac{5}{4}R_1 \rightarrow R_1 \left[\begin{array}{cccc|cccc|c} 5/4 & 1 & 0 & 0 & 5/4 & -1/4 & 1/2 & 0 & 30 \\ 0 & 1/5 & 0 & 1 & 0 & 1/5 & -1/5 & 0 & 20 \\ -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ -3 & -1 & 0 & 0 & 0 & 1 & 2 & 1 & 300 \end{array} \right]$$

$$\begin{aligned} -1/5R_1 + R_2 \rightarrow R_2 & \left[\begin{array}{cccc|cccc|c} 5/4 & 1 & 0 & 0 & 5/4 & -1/4 & 1/2 & 0 & 30 \\ -1/4 & 0 & 0 & 1 & -1/4 & 1/4 & -1/2 & 0 & 14 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ 17/4 & 0 & 0 & 0 & 5/4 & -3/4 & -5/2 & 1 & 330 \end{array} \right] \\ R_1 + R_4 \rightarrow R_4 & \end{aligned}$$

∴ max is 330
attained at
 $x_1 = 0, x_2 = 30$
 $x_3 = 50, x_4 = 14$