Dawson College: Linear Algebra:

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Bonus Assignment 1

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Use Cramer's rule solve the following system

$$\begin{aligned} x_1 &= 0 \\ 2x_1 + 2x_2 &= 1 \\ 3x_1 - x_2 - 3x_3 &= 2 \end{aligned} \iff Ax = b \quad \text{where } A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & -1 & -3 \end{bmatrix}$$

and $b = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

$\det A = -6$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & -1 & -3 \end{bmatrix} \Rightarrow \det A_1 = 0$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & -3 \end{bmatrix} \Rightarrow \det A_2 = -3$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 1 \\ 3 & -1 & 2 \end{bmatrix} \Rightarrow \det A_3 = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$= 1\,C_{11} + 0C_{12} + 0C_{13}$$

$$= 1(-1)^{1+1} \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}$$

$$= [4 + 1] = 5$$

$$\therefore \quad x_1 = \frac{\det A_1}{\det A} = \frac{0}{-6} = 0, \qquad x_2 = \frac{\det A_2}{\det A} = \frac{-3}{-6} = \frac{1}{2}$$

$$x_3 = \frac{\det A_3}{\det A} = \frac{5}{-6}$$

Question 2. *(5 marks)* Use the adjoint to find the inverse of the following matrix

$$B=\begin{bmatrix}2 & 3 & 0\\ 4 & 5 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

$$B^{-1}=\frac{1}{\det B}\,\mathrm{Adj}(B)=\frac{1}{\det B}\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^t$$

$$\det B = b_{31}C_{31}+b_{32}C_{32}+b_{33}C_{33}$$

$$=(1)C_{33}=(1)(-1)^{3+3}\begin{vmatrix}2 & 3\\ 4 & 5\end{vmatrix}=[10-12]=-2$$

$$C_{11}=(-1)^{1+1}\begin{vmatrix}5 & 0\\ 0 & 1\end{vmatrix}=5 \qquad C_{21}=(-1)^{2+1}\begin{vmatrix}3 & 0\\ 0 & 1\end{vmatrix}=-3$$

$$C_{12}=(-1)^{1+2}\begin{vmatrix}4 & 0\\ 0 & 1\end{vmatrix}=-4 \qquad C_{22}=(-1)^{2+2}\begin{vmatrix}2 & 0\\ 0 & 1\end{vmatrix}=2$$

$$C_{13}=(-1)^{1+3}\begin{vmatrix}4 & 5\\ 0 & 0\end{vmatrix}=0 \qquad C_{23}=(-1)^{2+3}\begin{vmatrix}2 & 3\\ 0 & 0\end{vmatrix}=0$$

$$C_{31}=(-1)^{3+1}\begin{vmatrix}3 & 0\\ 5 & 0\end{vmatrix}=0$$

$$C_{32}=(-1)^{3+2}\begin{vmatrix}2 & 0\\ 4 & 0\end{vmatrix}=0$$

$$C_{33}=(-1)^{3+3}\begin{vmatrix}2 & 3\\ 4 & 5\end{vmatrix}=-2$$

$$B^{-1}=\frac{1}{-2}\begin{bmatrix}5 & -4 & 0\\ -3 & 2 & 0\\ 0 & 0 & -2\end{bmatrix}^t$$

$$=\frac{1}{-2}\begin{bmatrix}5 & -3 & 0\\ -4 & 2 & 0\\ 0 & 0 & -2\end{bmatrix}$$

$$=\begin{bmatrix}-\frac{5}{2} & \frac{3}{2} & 0\\ 2 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$$