

## Bonus Assignment 2

You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (10 marks) Use the combinatorial definition of the determinant to compute the determinant of the following matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}.$$

*Solution.* Let  $S = \{1, 2, 3, 4\}$ .

Permutation of $S$	Total Number of Inversion	Parity of the Permutation	Signed Elementary Product
(1, 2, 3, 4)	0	even	$a_{11}a_{22}a_{33}a_{44}$
(1, 2, 4, 3)	1	odd	$-a_{11}a_{22}a_{34}a_{43}$
(1, 3, 2, 4)	1	odd	$-a_{11}a_{23}a_{32}a_{44}$
(1, 3, 4, 2)	2	even	$a_{11}a_{23}a_{34}a_{42}$
(1, 4, 2, 3)	2	even	$a_{11}a_{24}a_{32}a_{43}$
(1, 4, 3, 2)	3	odd	$-a_{11}a_{24}a_{33}a_{42}$
(2, 1, 3, 4)	1	odd	$-a_{12}a_{21}a_{33}a_{44}$
(2, 1, 4, 3)	2	even	$a_{12}a_{21}a_{34}a_{43}$
(2, 3, 1, 4)	2	even	$a_{12}a_{23}a_{31}a_{44}$
(2, 3, 4, 1)	3	odd	$-a_{12}a_{23}a_{34}a_{41}$
(2, 4, 1, 3)	3	odd	$-a_{12}a_{24}a_{31}a_{43}$
(2, 4, 3, 1)	4	even	$a_{12}a_{24}a_{33}a_{41}$
(3, 1, 2, 4)	2	even	$a_{13}a_{21}a_{32}a_{44}$
(3, 1, 4, 2)	3	odd	$-a_{13}a_{21}a_{34}a_{42}$
(3, 2, 1, 4)	3	odd	$-a_{13}a_{22}a_{31}a_{44}$
(3, 2, 4, 1)	4	even	$a_{13}a_{22}a_{34}a_{41}$
(3, 4, 1, 2)	4	even	$a_{13}a_{24}a_{31}a_{42}$
(3, 4, 2, 1)	5	odd	$-a_{13}a_{24}a_{32}a_{41}$
(4, 1, 2, 3)	3	odd	$-a_{14}a_{21}a_{32}a_{43}$
(4, 1, 3, 2)	4	even	$a_{14}a_{21}a_{33}a_{42}$
(4, 2, 1, 3)	4	even	$a_{14}a_{22}a_{31}a_{43}$
(4, 2, 3, 1)	5	odd	$-a_{14}a_{22}a_{33}a_{41}$
(4, 3, 1, 2)	5	odd	$-a_{14}a_{23}a_{31}a_{42}$
(4, 3, 2, 1)	6	even	$a_{14}a_{23}a_{32}a_{41}$

The determinant of  $A$  is the sum of all the signed elementary product of  $A$ .

$$\begin{aligned} \det(A) = & a_{11}a_{22}a_{33}a_{44} - a_{11}a_{22}a_{34}a_{43} - a_{11}a_{23}a_{32}a_{44} + a_{11}a_{23}a_{34}a_{42} + a_{11}a_{24}a_{32}a_{43} - a_{11}a_{24}a_{33}a_{42} - a_{12}a_{21}a_{33}a_{44} \\ & + a_{12}a_{21}a_{34}a_{43} + a_{12}a_{23}a_{31}a_{44} - a_{12}a_{23}a_{34}a_{41} - a_{12}a_{24}a_{31}a_{43} + a_{12}a_{24}a_{33}a_{41} + a_{13}a_{21}a_{32}a_{44} - a_{13}a_{21}a_{34}a_{42} \\ & - a_{13}a_{22}a_{31}a_{44} + a_{13}a_{22}a_{34}a_{41} + a_{13}a_{24}a_{31}a_{42} - a_{13}a_{24}a_{32}a_{41} - a_{14}a_{21}a_{32}a_{43} + a_{14}a_{21}a_{33}a_{42} \\ & + a_{14}a_{22}a_{31}a_{43} - a_{14}a_{22}a_{33}a_{41} - a_{14}a_{23}a_{31}a_{42} + a_{14}a_{23}a_{32}a_{41} \end{aligned}$$