

## Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Solve the following system of linear equations using Gauss-Jordan elimination.

$$\begin{aligned} 2x_1 + 2x_3 - x_4 &= 0 \\ -2x_1 - x_3 + 2x_4 &= 2 \\ 3x_1 - 3x_2 + 4x_3 - x_4 &= -1 \end{aligned}$$

$$\begin{bmatrix} 2 & 0 & 2 & -1 & 0 \\ -2 & 0 & -1 & 2 & 2 \\ 3 & -3 & 4 & -1 & -1 \end{bmatrix} \sim \begin{matrix} R_1 + R_2 \rightarrow R_2 \\ 2R_3 \end{matrix} \begin{bmatrix} 2 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 6 & -6 & 8 & -2 & -2 \end{bmatrix}$$

$$\sim \begin{matrix} -3R_1 + R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} 2 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & -6 & 2 & 1 & -2 \end{bmatrix}$$

$$\sim R_2 \leftrightarrow R_3 \begin{bmatrix} 2 & 0 & 2 & -1 & 0 \\ 0 & -6 & 2 & 1 & -2 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

$$\sim \begin{matrix} -2R_3 + R_1 \rightarrow R_1 \\ -2R_3 + R_2 \rightarrow R_2 \end{matrix} \begin{bmatrix} 2 & 0 & 0 & -3 & -4 \\ 0 & -6 & 0 & -1 & -6 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix} \sim \begin{matrix} \frac{1}{2}R_1 \\ \frac{1}{6}R_2 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{3}{2} & -2 \\ 0 & 1 & 0 & \frac{1}{6} & 1 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

Let  $x_4 = t$  ①

$$x_1 - \frac{3}{2}x_4 = -2$$

$$x_2 + \frac{1}{6}x_4 = 1$$

$$x_3 + x_4 = 2$$

$$\left. \begin{matrix} x_1 - \frac{3}{2}x_4 = -2 \\ x_2 + \frac{1}{6}x_4 = 1 \\ x_3 + x_4 = 2 \end{matrix} \right\} \text{sub } \textcircled{1} \Leftrightarrow \begin{matrix} x_1 - \frac{3}{2}t = -2 \\ x_2 + \frac{1}{6}t = 1 \\ x_3 + t = 2 \end{matrix}$$

∴ the general solution is

$$x_1 = -2 + \frac{3}{2}t$$

$$x_2 = 1 - \frac{1}{6}t$$

$$x_3 = 2 - t$$

$$x_4 = t$$