

## Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** Solve the following system of linear equations using Gaussian elimination and back substitution.

$$\begin{aligned} 2x_1 & & & - & x_4 & = & 0 \\ -2x_1 & & & - & x_3 & + & 2x_4 & = & 2 \\ 3x_1 & - & 3x_2 & + & 4x_3 & - & x_4 & = & -1 \end{aligned}$$

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ -2 & 0 & -1 & 2 & 2 \\ 3 & -3 & 4 & -1 & -1 \end{bmatrix} \sim R_1 + R_2 \rightarrow R_2 \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 2 & 2 \\ 2R_3 & 6 & -6 & 8 & -2 & -2 \end{bmatrix}$$

$$\sim -3R_1 + R_3 \rightarrow R_3 \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 2 \\ 0 & -6 & 8 & 1 & -2 \end{bmatrix}$$

$$\sim R_2 \leftrightarrow R_3 \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 0 & -6 & 8 & 1 & -2 \\ 0 & 0 & -1 & 1 & 2 \end{bmatrix}$$

Let  $x_4 = t$  and sub into

$$x_1 - \frac{1}{2}x_4 = 0$$

$$x_2 - \frac{4}{3}x_4 - \frac{1}{6}x_4 = \frac{1}{3}$$

$$x_3 - x_4 = -2$$

$$\sim \begin{aligned} & \frac{1}{2}R_1 \\ & -\frac{1}{6}R_2 \\ & -R_3 \end{aligned} \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{4}{3} & -\frac{1}{6} & \frac{1}{3} \\ 0 & 0 & 1 & -1 & -2 \end{bmatrix}$$

we get

$$x_1 - \frac{1}{2}t = 0$$

$$x_2 - \frac{4}{3}x_3 - \frac{1}{6}t = \frac{1}{3} \quad (2)$$

$$x_3 - t = -2$$

So  $x_1 = \frac{1}{2}t$

$x_3 = -2 + t$  (1) and sub (1) into (2)

$$x_2 - \frac{4}{3}(-2 + t) - \frac{1}{6}t = \frac{1}{3}$$

$$x_2 + \frac{8}{3} - \frac{4t}{3} - \frac{1}{6}t = \frac{1}{3}$$

$$x_2 = -\frac{7}{3} + \frac{9}{6}t$$

∴ the general solution is  $(x_1, x_2, x_3, x_4) = (\frac{1}{2}t, -\frac{7}{3} + \frac{9}{6}t, -2 + t, t)$