

## Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let  $S = \{1, 2, 3, 4, 5, 6\}$ .

- a. (1 mark) Give two permutations of the set  $S$ .  $(2, 1, 3, 4, 5, 6)$ ,  $(6, 1, 2, 3, 4, 5)$

- b. (2 marks) Is  $(2, 1, 3, 4, 5, 6)$  a permutation of the set  $S$ , justify.

Yes it is a permutation since there is no omission and no repetition.

- c. (2 marks) Determine the parity of the permutation  $(5, 2, 1, 3, 6, 4)$  of the set  $S$ .

$$\# \text{ of inversions} = 4 + 1 + 0 + 0 + 1 = 6$$

Question 2. (5 marks) If

$$A = \begin{bmatrix} 2 & 1 & -2 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$\therefore$  even

then compute  $\det(A)$  using a cofactor expansion.

$$\det A = a_{12} C_{12} + a_{22} C_{22} + a_{32} C_{32} + a_{42} C_{42} + a_{52} C_{52}$$

$$= 1 C_{12} + 0 C_{22} + 0 C_{32} + 0 C_{42} + 0 C_{52}$$

$$= 1(-1)^{1+2} \begin{vmatrix} 3 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{vmatrix} = -1 [a_{31} C_{31} + a_{32} C_{32} + a_{33} C_{33} + a_{34} C_{34}]$$

$$= -1 [0 C_{31} + 0 C_{32} + 0 C_{33} + 3 C_{34}]$$

$$= -1 \cdot 3 \cdot (-1)^{3+4} \begin{vmatrix} 3 & 1 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 3 [a_{31} C_{31} + a_{32} C_{32} + a_{33} C_{33}]$$

$$= 3 [0 C_{31} + 0 C_{32} + 1 C_{33}]$$

$$= 3 \cdot 1 \cdot (-1)^{3+3} \begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 3 [3(1) - (1)(0)]$$

$$= 9$$