

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let $S = \{1, 2, 3, 4, 5\}$.

- a. (1 mark) Give two permutations of the set S . $(5, 4, 3, 2, 1), (3, 2, 1, 4, 5)$

- b. (2 marks) Is $(5, 2, 1, 3, 4, 5)$ a permutation of the set S , justify.

Not a permutation since 5 is repeated.

- c. (2 marks) Determine the parity of the permutation $(5, 2, 1, 3, 4)$ of the set S . # of inversions = $4 + 1 + 0 + 0 + 0 = 5$

Question 2. (5 marks) If

$$A = \begin{bmatrix} 2 & 1 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

∴ odd

then compute $\det(A)$ using a cofactor expansion.

$$\det A = a_{13} C_{13} + a_{23} C_{23} + a_{33} C_{33} + a_{43} C_{43} + a_{53} C_{53}$$

$$= -2 C_{13} + 0 C_{23} + 0 C_{33} + 0 C_{43} + 0 C_{53}$$

$$= -2(-1)^{1+3} \begin{vmatrix} 3 & 2 & 0 & 0 \\ 0 & 2 & -2 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{vmatrix} = -2 [a_{31} C_{31} + a_{32} C_{32} + a_{33} C_{33} + a_{34} C_{34}]$$

$$= -2 [0 C_{31} + 0 C_{32} + 0 C_{33} + 3 C_{34}]$$

$$= -6(-1)^{3+4} \begin{vmatrix} 3 & 2 & 0 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 6 [a_{31} C_{31} + a_{32} C_{32} + a_{33} C_{33}]$$

$$= 6 [0 C_{31} + 0 C_{32} + 1 C_{33}]$$

$$= 6(-1)^{3+3} \begin{vmatrix} 3 & 2 \\ 0 & 2 \end{vmatrix}$$

$$= 6 [3(2) - (2)(0)]$$

$$= 6(6)$$

$$= 36$$