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Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let $A = (1,2,0)$, $B = (0,-2,3)$, $P = (1,1,-1)$, $\mathbf{u} = (3,-1,2)$ and $\mathbf{v} = (4,1,-3)$.

  a. *(2 marks)* Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

  b. *(2 marks)* Find a unit vector orthogonal to both $\mathbf{u}$ and $\mathbf{v}$

  c. *(6 marks)* Using projections find the distance from point P to the line that passes through the points A and B.

a) $\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$

$3(4) + (-1)(1) + 2(-3) = \sqrt{3^2+(-1)^2+2^2}\sqrt{4^2+1^2+(-3)^2} \cos\theta$

$5 = \sqrt{14}\sqrt{26} \cos\theta$

$\dfrac{5}{\sqrt{14}\sqrt{26}} = \cos\theta$
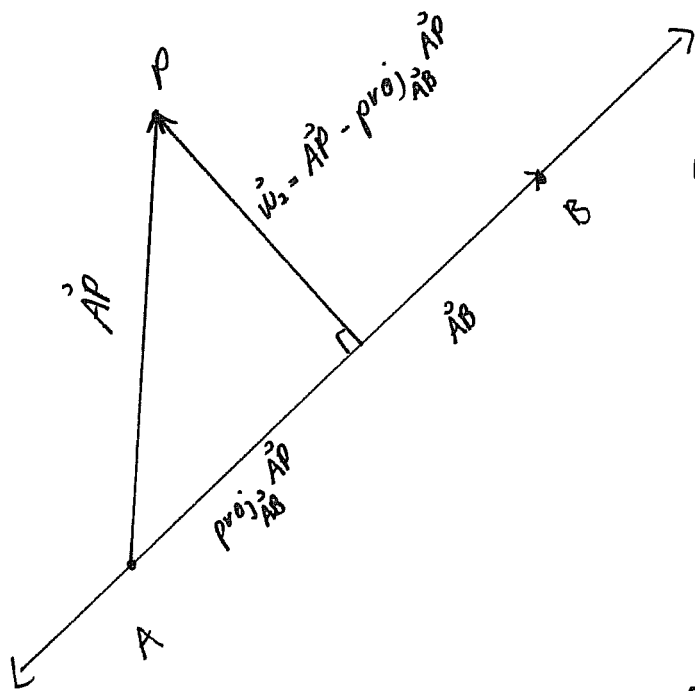
$\dfrac{5}{2\sqrt{91}} = \cos\theta$

$\arccos\left(\dfrac{5}{2\sqrt{91}}\right) = \theta$

b) $\vec{u} \times \vec{v} = \left(\begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix}, -\begin{vmatrix} 3 & 4 \\ 2 & -3 \end{vmatrix}, \begin{vmatrix} 3 & 4 \\ -1 & 1 \end{vmatrix}\right)$

$\begin{matrix} 3 & 4 \\ -1 & 1 \\ 2 & -3 \end{matrix} = (1, +17, 7)$

$\therefore \dfrac{\vec{u} \times \vec{v}}{\|\vec{u} \times \vec{v}\|} = \dfrac{(1, 17, 7)}{\sqrt{1^2+17^2+(7)^2}}$

$= \dfrac{(1, 17, 7)}{\sqrt{339}}$

c)

$\vec{AP} = P - A = (1,1,-1) - (1,2,0) = (0,-1,-1)$

$\vec{AB} = B - A = (0,-2,3) - (1,2,0) = (-1,-4,+3)$

$\vec{w_2} = (0,-1,-1) - \dfrac{(0,-1,-1)\cdot(-1,-4,3)}{(-1,-4,3)\cdot(-1,-4,3)}(-1,-4,3)$

$= (0,-1,-1) - \dfrac{0(-1) + (-1)(-4) + (-1)(3)}{(-1)(-1) + (-4)(-4) + 3(3)}(-1,-4,3)$

$= (0,-1,-1) - \dfrac{1}{26}(-1,-4,3)$

$= \left(\dfrac{1}{26}, \dfrac{-22}{26}, \dfrac{-29}{26}\right)$

$\therefore \text{distance} = \|w_2\| = \sqrt{\left(\dfrac{1}{26}\right)^2 + \left(\dfrac{-22}{26}\right)^2 + \left(\dfrac{-29}{26}\right)^2} = \sqrt{\dfrac{1326}{26^2}}$

$= \sqrt{\dfrac{51}{26}}$