

Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let $A = (0, 2, -1)$, $B = (3, 0, -3)$, $P = (-1, 1, -1)$, $\mathbf{u} = (-3, 2, 1)$ and $\mathbf{v} = (2, 1, -5)$.

- a. (2 marks) Find the angle between \mathbf{u} and \mathbf{v} .
- b. (2 marks) Find a unit vector orthogonal to both \mathbf{u} and \mathbf{v} .
- c. (6 marks) Using projections find the distance from point P to the line that passes through the points A and B .

a) $\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta$

$$-3(2) + 2(1) + 1(-5) = \sqrt{(-3)^2 + 2^2 + 1^2} \sqrt{2^2 + 1^2 + (-5)^2} \cos \theta$$

$$-9 = \sqrt{14} \sqrt{30} \cos \theta$$

$$\frac{-9}{\sqrt{14}\sqrt{30}} = \cos \theta$$

$$\frac{-9}{\sqrt{7 \cdot 2 \cdot 2 \cdot 5 \cdot 3}} = \cos \theta$$

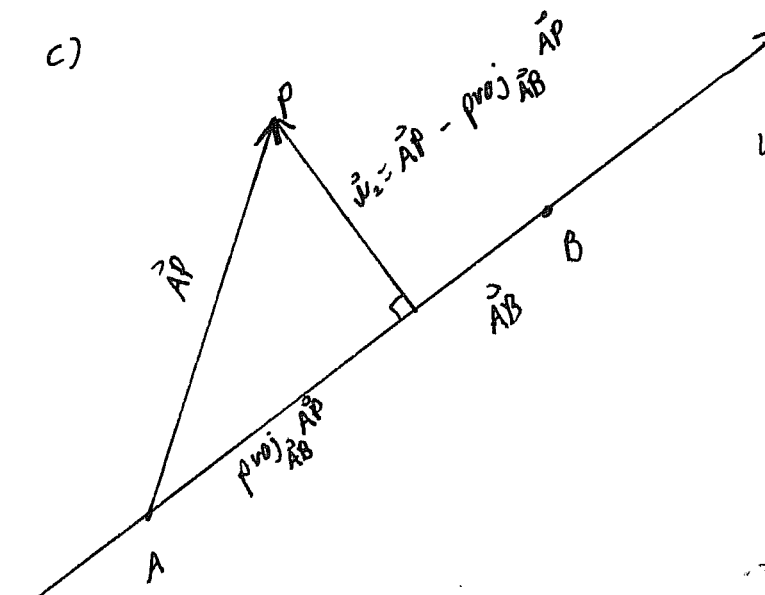
$$\frac{-9}{2\sqrt{105}} = \cos \theta$$

$$\arccos\left(\frac{-9}{2\sqrt{105}}\right) = \theta$$

b) $\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 1 \\ 2 & 1 & -5 \end{vmatrix} = (-11, -13, -7)$

$$\therefore \frac{\vec{u} \times \vec{v}}{\|\vec{u} \times \vec{v}\|} = \frac{(-11, -13, -7)}{\sqrt{11^2 + 13^2 + 7^2}} = \frac{(-11, -13, -7)}{\sqrt{339}}$$

c)



$$\vec{AP} = P - A = (-1, 1, -1) - (0, 2, -1) = (-1, -1, 0)$$

$$\vec{AB} = B - A = (3, 0, -3) - (0, 2, -1) = (3, -2, -2)$$

$$\vec{w}_2 = (-1, -2, 0) - \frac{(-1, -1, 0) \cdot (3, -2, -2)}{(3, -2, -2) \cdot (3, -2, -2)} (3, -2, -2)$$

$$= (-1, -2, 0) - \frac{-1}{9+4+4} (3, -2, -2)$$

$$= (-1, -2, 0) - \frac{-1}{17} (3, -2, -2)$$

$$= \left(\frac{-14}{17}, \frac{-19}{17}, \frac{-2}{17}\right)$$

$$\therefore \text{distance} = \|\vec{w}_2\| = \sqrt{\left(\frac{-14}{17}\right)^2 + \left(\frac{-19}{17}\right)^2 + \left(\frac{-2}{17}\right)^2} = \sqrt{\frac{561}{17^2}} = \sqrt{\frac{33}{17}}$$