

Quiz 9

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (10 marks) Consider the system

$$\begin{aligned} x_1 &+ x_3 + x_4 = 5 \\ 2x_1 + 6x_2 - 4x_3 + 2x_4 &= 4 \\ -3x_1 + 2x_2 - 2x_3 &= -2 \end{aligned}$$

Solve the system by Gauss-Jordan elimination and find the particular solution of the system with $x_2 = 3$.

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 5 \\ 2 & 6 & -4 & 2 & 4 \\ -3 & 2 & -2 & 0 & -2 \end{bmatrix} \sim \begin{array}{l} -2R_1 + R_2 \rightarrow R_2 \\ 3R_1 + R_3 \rightarrow R_3 \end{array} \begin{bmatrix} 1 & 0 & 1 & 1 & 5 \\ 0 & 6 & -6 & 0 & -6 \\ 0 & 2 & 1 & 3 & 13 \end{bmatrix}$$

$$\text{if } x_2 = 3$$

$$3 = 4 - t$$

$$t = 1$$

$$\begin{aligned} \therefore (x_1, x_2, x_3, x_4) \\ = (0, 3, 4, 1). \end{aligned}$$

$$\sim \frac{1}{6}R_2 \begin{bmatrix} 1 & 0 & 1 & 1 & 5 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 2 & 1 & 3 & 13 \end{bmatrix}$$

$$\sim \begin{array}{l} -2R_2 + R_3 \rightarrow R_3 \end{array} \begin{bmatrix} 1 & 0 & 1 & 1 & 5 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 3 & 3 & 15 \end{bmatrix}$$

$$\sim \frac{1}{3}R_3 \begin{bmatrix} 1 & 0 & 1 & 1 & 5 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 5 \end{bmatrix}$$

$$\sim \begin{array}{l} -R_3 + R_1 \rightarrow R_1 \\ R_3 + R_2 \rightarrow R_2 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 1 & 5 \end{bmatrix}$$

$$\text{let } x_4 = t$$

$$x_1 = 0$$

$$x_2 = 4 - t$$

$$x_3 = 5 - t$$