

## Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

## Question 1.

a. (5 marks) Solve by Cramer's rule

$$\begin{aligned} 7x + 5y &= 9 \\ 5x + 7y &= 3 \end{aligned}$$

$$\Leftrightarrow Ax = b \text{ where } A = \begin{bmatrix} 7 & 5 \\ 5 & 7 \end{bmatrix} \text{ and } b = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

b. (5 marks) Find the matrix A such that

$$(A^t - 5I)^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 9 & 5 \\ 3 & 7 \end{bmatrix}, A_2 = \begin{bmatrix} 7 & 9 \\ 5 & 3 \end{bmatrix}$$

where I is the  $2 \times 2$  identity matrix.

$$\det A_1 = 9 \cdot 7 - 5 \cdot 3 = 48$$

$$\det A_2 = 7 \cdot 3 - 9 \cdot 5 = -24$$

$$\det A = 49 - 25 = 24$$

$$\therefore x = \frac{\det A_1}{\det A} = \frac{48}{24} = 2$$

$$y = \frac{\det A_2}{\det A} = \frac{-24}{24} = -1$$

$$(A^t - 5I)^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$A^t - 5I = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^{-1}$$

$$A^t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 5 & -1 \\ 1 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 1 \\ -1 & 5 \end{bmatrix}$$