

Test 1

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (10 marks) Solve the following system by Gauss-Jordan elimination:

$$\begin{array}{rccccrcr} 3x_1 & + & 2x_2 & + & 2x_3 & & = & 1 \\ 4x_1 & + & 1x_2 & - & 3x_3 & + & x_4 & = & 2 \\ 7x_1 & + & 3x_2 & - & x_3 & + & x_4 & = & 3 \end{array}$$

In addition, if the solution is not unique give two particular solutions.

Question 2. (5 marks) Consider the matrices:

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & -1 \\ 1 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 & -1 \\ 0 & -4 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

Then compute $\text{tr}(AA^t - 2B)$, if possible.

Question 3. (5 marks) Solve for the matrix X if

$$X \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 4 & 4 & 6 \end{bmatrix}$$

Question 4. (10 marks) Solve the following system using the inverse of the coefficient matrix.

$$x_1 + 2x_2 + 3x_3 = -1$$

$$2x_1 + 3x_2 + 4x_3 = 0$$

$$1x_1 + 2x_2 + 4x_3 = 1$$

Question 5. Express

$$A = \begin{bmatrix} 3 & 4 \\ 5 & -2 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ 11 & 6 \end{bmatrix}, C = \begin{bmatrix} 11 & 6 \\ 3 & 4 \end{bmatrix}$$

Find the elementary matrices E_1 and E_2 (if possible) such that

- a. (2 marks) $E_1A = B$
- b. (2 marks) $E_2A = C$

Question 6. (3 marks) Show that if A, D are invertible and $ABD = ACD$ then $B = C$.

Question 7. (3 marks) Find A^{-1} if $A^3 + 3A - I = 0$.

Question 8. (5 marks) If D is an $n \times n$ diagonal matrix, A is an $n \times n$ symmetric matrix and B is an $n \times n$ matrix then show that $B^tB + 2A + D - I$ is symmetric.

Question 9. (5 marks) For which value(s) of the constant a and b does the system

$$\begin{aligned}x + y &= 1 \\4x + a^2y &= b\end{aligned}$$

have no solutions? Exactly one solution? Infinitely many solutions? Justify.

Bonus Question. (3 marks) Prove: If $A^5 = 0$ then

$$(I - A)^{-1} = I + A + A^2 + A^3 + A^4.$$