NAME: SOLUTIONS

QUIZ 3

Calculus 1 for Electrotech (201-NYA-DW 07)
Instructor: Emilie Richer
Date: February 5th 2010

[QUESTION 1] *(12 marks)*
Evaluate the limits. (If they do not exist write DNE, and determine whether they tend to + or – ∞)

1. $\lim\limits_{x \to -\infty} \dfrac{2x^5 - 3}{-3x^4 + 2x^2}$

$$= \lim_{x \to -\infty} \frac{\frac{2x^5}{x^4} - \frac{3}{x^4}}{\frac{-3x^4}{x^4} + \frac{2x^2}{x^4}} = \lim_{x \to -\infty} \frac{2x - \cancel{\frac{3}{x^4}}\nearrow 0}{-3 + \cancel{\frac{2}{x^2}}\nearrow 0} = \lim_{x \to -\infty} \frac{2x}{-3} \quad \text{DNE}$$

$\boxed{\text{tends to } \infty}$

2. $\lim\limits_{x \to 9} \dfrac{\sqrt{x} - 3}{x^2 - 81}$

$$= \lim_{x \to 9} \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)}{(x^2 - 81)(\sqrt{x} + 3)}$$

$$= \lim_{x \to 9} \frac{\cancel{x - 9}}{(x+9)\cancel{(x-9)}(\sqrt{x}+3)}$$

$$= \lim_{x \to 9} \frac{1}{(x+9)(\sqrt{x}+3)} = \lim_{x \to 9} \frac{1}{(9+9)(3+3)} = \frac{1}{18(6)} = \boxed{\frac{1}{108}}$$

3. $\lim\limits_{x \to \infty} \dfrac{1 + 2x^2 - 2x^7}{-2x^4 + 6x^7}$

$$= \lim_{x \to \infty} \frac{\frac{1}{x^7} + \frac{2x^2}{x^7} - \frac{2x^7}{x^7}}{\frac{-2x^4}{x^7} + \frac{6x^7}{x^7}} = \lim_{x \to \infty} \frac{\cancel{\frac{1}{x^7}}\nearrow 0 + \cancel{\frac{2}{x^5}}\nearrow 0 - 2}{-\cancel{\frac{2}{x^3}}\nearrow 0 + 6}$$

$$= \frac{-2}{6} = \boxed{-\frac{1}{3}}$$

[QUESTION 2] *(3 marks)*
State the three conditions that must be satisfied in order for a function *f*(x) to be continous at x = b.

(1) $f(b)$ EXISTS

(2) $\lim_{x \to b} f(x)$ EXISTS

(3) $f(b) = \lim_{x \to b} f(x)$

[QUESTION 3] (5 marks)
Determine whether the following function is *f*(x) continuous at x = 3. (Justify your answer using the three conditions stated in question 2).

$$f(x) = \begin{cases} x - 2 & x > 3 \\ 1 & x = 3 \\ -x^2 + 2x + 4 & x < 3 \end{cases}$$

(1) $f( )$ EXISTS ; $f(3) = 1$

(2) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} -x^2 + 2x + 4$

$= -(3)^2 + 2(3) + 4$

$= -9 + 6 + 4 = 1$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x - 2$

$= 3 - 2 = 1$

so $\lim_{x \to 3} f(x) = 1$ EXISTS

(3) $f(3) = \lim_{x \to 3} f(x)$

(BOTH ARE 1)

f is CONTINUOUS at $x = 3$.

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QUIZ 3B

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[QUESTION 1] *(12 marks)*
Evaluate the limits. (If they do not exist write DNE, and determine whether they tend to + or – ∞)

1. $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x^2 - 16}$

$$= \lim_{x\to 4} \frac{\sqrt{x}-2}{x^2-16}\,\frac{(\sqrt{x}+2)}{(\sqrt{x}+2)}$$

$$= \lim_{x\to 4} \frac{\cancel{x-4}}{(x+4)\cancel{(x-4)}(\sqrt{x}+2)}$$

$$= \lim_{x\to 4} \frac{1}{(x+4)(\sqrt{x}+2)}$$

$$= \frac{1}{(4+4)(\sqrt{4}+2)}$$

$$= \frac{1}{8(4)} = \boxed{\frac{1}{32}}$$

2. $\lim_{x \to \infty} \frac{2x^5 - 3}{-3x^4 + 2x^2}$

$$= \lim_{x\to\infty} \frac{\frac{2x^5}{x^4} - \frac{3}{x^4}}{\frac{-3x^4}{x^4} + \frac{2x^2}{x^4}} = \lim_{x\to\infty} \frac{2x - \cancel{\frac{3}{x^4}}^{\,0}}{-3 + \cancel{\frac{2}{x^2}}^{\,0}} = \lim_{x\to\infty} \frac{2x}{-3} \text{ DNE}$$

$$\boxed{\xrightarrow[\text{tends to}]{} -\infty}$$

3. $\lim_{x \to -\infty} \frac{1 + 2x^2 - 3x^7}{-3x^4 + 7x^7}$

$$= \lim_{x\to-\infty} \frac{\frac{1}{x^7} + \frac{2x^2}{x^7} - \frac{3x^7}{x^7}}{\frac{-3x^4}{x^7} + \frac{7x^7}{x^7}} = \lim_{x\to-\infty} \frac{\cancel{\frac{1}{x^7}}^{\,0} + \cancel{\frac{2}{x^5}}^{\,0} - 3}{\cancel{\frac{-3}{x^3}}^{\,0} + 7} = \boxed{\frac{-3}{7}}$$

[QUESTION 2] *(3 marks)*
State the three conditions that must be satisfied in order for a function *f*(x) to be continous at x = b.

(1) $f(b)$ EXISTS

(2) $\lim_{x \to b} f(x)$ EXISTS

(3) $f(b) = \lim_{x \to b} f(x)$

[QUESTION 3] (5 marks)
Determine whether the following function is *f*(x) continuous at x = 1. (Justify your answer using the three conditions stated in question 2).

$$f(x) = \begin{cases} x - 2 & x > 1 \\ -1 & x = 1 \\ -x^2 + 2x - 2 & x < 1 \end{cases}$$

(1) $f(1)$ EXISTS ; $f(1) = -1$

(2) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} -x^2 + 2x - 2$

$= -1 + 2 - 2 = -1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x - 2$

$= 1 - 2 = -1$

SO $\lim_{x \to 1} f(x)$ EXISTS & IS EQUAL to -1

(3) $f(1) = \lim_{x \to 1} f(x)$ (both are -1)

SO $f(x)$ IS CONTINUOUS.