NAME: SOLUTIONS

QUIZ 4

Calculus 1 for Electrotech (201-NYA-50 07)
Instructor: Emilie Richer
Date: February 12th 2010

For the following questions YOU MUST USE THE LIMIT METHOD for finding derivatives.
Remember to use PROPER LIMIT NOTATION

[QUESTION 1] *(4 marks)*
Find the slope of the tangent line to the curve $f(x) = x^2 - 3$ when $x = 2$.

$$\text{slope} = \lim_{h\to 0} \frac{[(x+h)^2-3] - (x^2-3)}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-3-x^2+3}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$$

AT $x=2$ slope is $2(2) = \boxed{4}$

[QUESTION 2] *(5 marks)*
Find the derivative of the function $f(x) = \sqrt{x} - 4$

$$f'(x) = \lim_{h\to 0} \frac{(\sqrt{x+h}-4) - (\sqrt{x}-4)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

$$= \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$f'(x) = \boxed{\frac{1}{2\sqrt{x}}}$$

[QUESTION 3] (6 marks)
Find the **points** (x **and** y coordinates) on the curve $f(x) = \dfrac{1}{3x+1}$

where the tangent line to the curve f(x) is parallel to the line y = –3x + 3.

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{3(x+h)+1} - \frac{1}{3x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3x+1 - (3(x+h)+1)}{(3(x+h)+1)(3x+1)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3x+1-3x-3h-1}{(3x+3h+1)(3x+1)}}{h}$$

$$= \lim_{h \to 0} \frac{-3\cancel{h}}{(3x+3h+1)(3x+1)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{-3}{(3x+1)(3x+1)} = \frac{-3}{(3x+1)^2}$$

we need the points where this slope is -3

$$\frac{-3}{(3x+1)^2} = -3$$

$$-3 = -3(3x+1)^2$$

$$-3 = -3(9x^2+6x+1)$$

$$1 = 9x^2+6x+1$$

$$0 = 9x^2+6x$$

$$0 = 3x(3x+2)$$

$$x=0 \ \& \ x = -2/3$$

y values

AT $x = 0$ $\quad f(0) = \frac{1}{3(0)+1} = 1$

$\boxed{\text{POINT } (0,1)}$

AT $x = -2/3$ $\quad f(-2/3) = \frac{1}{3(-2/3)+1} = -1$

$\boxed{\text{POINT } (-2/3, -1)}$

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QUIZ 4B

Calculus 1 for Electrotech (201-NYA-50 07)
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Date: February 12th 2010

For the following questions YOU MUST USE THE LIMIT METHOD for finding derivatives.
Remember to use PROPER LIMIT NOTATION

[QUESTION 1] *(4 marks)*
Find the slope of the tangent line to the curve $f(x) = x^2 - 2$ when $x = 5$.

$$\text{SLOPE OF TANGENT} = \lim_{h\to 0} \frac{[(x+h)^2-2] - (x^2-2)}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-2-x^2+2}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$$

AT $x=5$ slope $= 2(5) = \boxed{10}$

[QUESTION 2] *(5 marks)*
Find the derivative of the function $f(x) = \sqrt{x} - 2$

$$f'(x) = \lim_{h\to 0} \frac{(\sqrt{x+h}-2)-(\sqrt{x}-2)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

$$= \frac{1}{\sqrt{x}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

[QUESTION 3] (6 marks)
Find the **point** (x **and** y coordinate) on the curve $f(x) = \dfrac{1}{2x+1}$

where the tangent line to the curve f(x) is parallel to the line $y = -2x + 3$.

SLOPE OF TANGENT LINE $= \lim\limits_{h \to 0} \dfrac{\dfrac{1}{2(x+h)+1} - \dfrac{1}{2x+1}}{h}$

$$= \lim_{h \to 0} \frac{2x+1 - (2x+2h+1)}{(2(x+h)+1)(2x+1)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{(2(x+h)+1)(2x+1)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{-2}{(2x+1)(2x+1)}$$

Since the tangent line must be parallel to $y = -2x + 3$ it must have slope -2

$$\frac{-2}{(2x+1)(2x+1)} = -2$$

$\Rightarrow \quad -2 = -2(2x+1)(2x+1)$

$\Rightarrow \quad 1 = (2x+1)(2x+1)$

$\Rightarrow \quad 1 = 4x^2 + 4x + 1$

$\Rightarrow \quad 0 = 4x^2 + 4x$

$\Rightarrow \quad 0 = 4x(x+1)$

$x = 0$ & $x = -1$

y - values

AT $x = 0$ $\quad f(0) = \dfrac{1}{2(0)+1} = 1$

$\boxed{\text{POINT } (0, 1)}$

AT $x = -1$ $\quad f(-1) = \dfrac{1}{2(-1)+1} = -1$

$\boxed{\text{POINT } (-1, -1)}$