

NAME: SOLUTIONS

### QUIZ 7

Dawson College

Course Code: 201-NYA-05 S07

Date: Mar 12th 2010

Instructor: E. Richer

#### Question 1. (5 marks)

Find  $\frac{d^2y}{dx^2} (y'')$  by implicit differentiation. Your final answer must only contain  $x$ 's and  $y$ 's.

①

$$\begin{aligned} xy + e^y &= -x \\ y + xy' + e^y y' &= -1 \\ y'(x + e^y) &= -1 - y \\ y' &= \frac{-1 - y}{x + e^y} \end{aligned}$$

② 
$$y'' = \frac{-y'(x + e^y) - (1 + e^y y')(-1 - y)}{(x + e^y)^2}$$

③ substitute  $y' = \frac{-1 - y}{x + e^y}$  into  $y''$

$$y'' = \frac{-(-1 - y)(x + e^y) - (1 + e^y \left(\frac{-1 - y}{x + e^y}\right))(-1 - y)}{(x + e^y)^2}$$

$$= \frac{(1 + y) - \frac{(x + e^y - e^y - ye^y)(-1 - y)}{(x + e^y)}}{(x + e^y)^2}$$

$$= \boxed{\frac{(1 + y)}{(x + e^y)^2} + \frac{(1 + y)(x - ye^y)}{(x + e^y)^3}}$$

#### Question 2. (5 marks)

Find  $f'''(x)$ .

Hint: Simplify your answer for  $f''(x)$  to make it easier to compute  $f'''(x)$ .

$$f(x) = \ln(x^2 + 1)$$

$$f'(x) = \frac{1}{x^2 + 1} (2x) = \frac{2x}{x^2 + 1}$$

$$f''(x) = \frac{2(x^2 + 1) - 2x(2x)}{(x^2 + 1)^2}$$

$$= \frac{2x^2 + 2 - 4x^2}{(x^2 + 1)^2}$$

$$= (-2x^2 + 2)(x^2 + 1)^{-2}$$

(PRODUCT RULE)  $\rightarrow$

$$f'''(x) = (-4x)(x^2 + 1)^{-2}$$

$$= \boxed{\frac{-4x}{(x^2 + 1)^2} - \frac{4x(-2x^2 + 2)}{(x^2 + 1)^3}}$$

Simplified:

$$= \frac{-4x(x^2 + 1) - (-8x^3 + 8x)}{(x^2 + 1)^3}$$

$$= \frac{-4x^3 - 4x + 8x^3 - 8x}{(x^2 + 1)^3} = \frac{4x^3 - 12x}{(x^2 + 1)^3}$$

$$= \frac{4x(x^2 - 3)}{(x^2 + 1)^3}$$

NAME: \_\_\_\_\_

### QUIZ 7b

Dawson College

Course Code: 201-NYA-05 S07

Date: Mar 12th 2010

Instructor: E. Richer

#### Question 1. (5 marks)

Find  $\frac{d^2y}{dx^2}$  ( $y''$ ) by implicit differentiation.  
x's and y's.

Your final answer must only contain

$$xy + e^y = -x$$

see Quiz 7

#### Question 2. (5 marks)

Find  $f'''(x)$ .

Hint: Simplify your answer for  $f''(x)$  to make it easier to compute  $f'''(x)$ .

$$f(x) = \ln(x^2 - 1)$$

$$f'(x) = \frac{1}{x^2 - 1} \cdot 2x$$

$$f''(x) = \frac{2(x^2 - 1) - 2x(2x)}{(x^2 - 1)^2}$$

$$= \frac{2x^2 - 2 - 4x^2}{(x^2 - 1)^2}$$

$$= \frac{-2x^2 - 2}{(x^2 - 1)^2}$$

By QUOTIENT RULE  $\rightarrow$

$$f'''(x) = \frac{-4x(x^2 - 1)^2 - 2(x^2 - 1)(2x)(-2x^2 - 2)}{(x^2 - 1)^4}$$

$$= \frac{-4x}{(x^2 - 1)^2} - \frac{4x(-2x^2 - 2)}{(x^2 - 1)^3}$$

Simplified

$$= \frac{-4x(x^2 - 1) + 8x^3 + 8x}{(x^2 - 1)^3}$$

$$= \frac{-4x^3 + 4x + 8x^3 + 8x}{(x^2 - 1)^3}$$

$$= \frac{4(x^3 + 3x)}{(x^2 - 1)^3} = \frac{4x(x^2 + 3)}{(x^2 - 1)^3}$$