SOLUTIONS TO TEST 2
NYA ELECTROTECH
WINTER 2010

1 (a) $f(x) = \log_5(xe^x)$

$$\boxed{f'(x) = \frac{1}{\ln 5}\left(\frac{1}{xe^x}\right)(e^x + e^x x)}$$

(b) $f(x) = \arctan(\ln x)$

$$\boxed{f'(x) = \frac{1}{1+(\ln x)^2}\left(\frac{1}{x}\right)}$$

(c) $f(x) = \cos(3^{4x})$

$$\boxed{f'(x) = -\sin(3^{4x})\ln 3\, 3^{4x}\, 4}$$

(d) $f(x) = \ln(\arcsin x^3)$

$$\boxed{f'(x) = \frac{1}{\arcsin x^3}\left(\frac{1}{\sqrt{1-x^6}}\right)(3x^2)}$$

(e) $$f(x) = \frac{\frac{-1}{\sqrt{1-x^2}}4^x - \ln 4\, 4^x \arccos x}{(4^x)^2}$$

$$= \frac{-4^x\left(1 + \ln 4(\arccos x)\sqrt{1-x^2}\right)}{\sqrt{1-x^2}\,(4^x)^2}$$

$$= \boxed{\frac{-\left(1 + \ln 4(\arccos x)\sqrt{1-x^2}\right)}{\left(\sqrt{1-x^2}\right)4^x}}$$

(2) $g'(t) = 12t^2 - 12t - 24$
$= 12(t^2 - t - 2)$
$= 12\,(t-2)(t+1)$

$g'(t) = 0$ WHEN $t = 2$ & $t = -1$

INTERVALS	$(-\infty, -1)$	$(-1, 2)$	$(2, \infty)$
TEST pt.	-2	0	3
sign of g'	$+$	$-$	$+$
behavior of g	↗	↘	↗

MAX AT $x = -1$ $Y = 4(-1)^3 - 6(-1)^2 - 24(-1) + 84$
$= -4 - 6 + 24 + 84 = 98$

$\boxed{\text{MAX } (-1, 98)}$

MIN AT $x = 2$ $Y = 4(2)^3 - 6(2)^2 - 24(2) + 84$
$= 32 - 24 - 48 + 84$
$= 44$

$\boxed{\text{MIN } (2, 44)}$

(3) $f(x) = x(x-4)^3$
$f'(x) = (x-4)^3 + 3(x-4)^2 x$
$= (x-4)^2(x - 4 + 3x)$
$= (x-4)^2(4x-4)$
$f''(x) = 2(x-4)(4x-4) + 4(x-4)^2$
$= 2(x-4)(4x - 4 + 2(x-4))$
$= 2(x-4)(6x - 12)$

$f''(x) = 0$ WHEN $x = 4$ & $x = 2$

INTERVAL	$(-\infty, 2)$	$(2, 4)$	$(4, \infty)$
TEST pt	0	3	5
sign OF f''	+	—	+
BEHAVIOR OF f	$\cup$	$\cap$	$\cup$

INFL pts AT $x = 2$ $\quad Y = 2(2-4)^3 = -16$

AT $x = 4$ $\quad Y = 0$

$\boxed{\begin{array}{c}(2, -16) \\ (4, 0)\end{array}}$

④ $y = (\tan x)^{\sin x}$

$\ln y = \ln(\tan x^{\sin x})$

$\ln y = (\sin x)(\ln \tan x)$

$\frac{1}{y} y' = \cos x \ln \tan x + \frac{1}{\tan x} \sec^2 x \sin x$

$y' = y \left[\cos x \ln \tan x + \frac{\sec^2 x \sin x}{\tan x}\right]$

$$\boxed{y' = (\tan x)^{\sin x} \left[\cos x \ln \tan x + \frac{\sec^2 x \sin x}{\tan x}\right]}$$

⑤ $\ln Y = \ln\left(\frac{\sin^2 x \, e^{3x} \tan x}{x^5 \sqrt{x} \, \operatorname{Arctan} x}\right)$

using properties of 'ln'

$\ln Y = 2 \ln \sin x + 3x + \ln \tan x - 5 \ln \quad - \frac{1}{2} \ln x - \ln \operatorname{Arctan} x$

$\frac{1}{Y} Y' = \frac{2}{\sin x} \cos x + 3 + \frac{1}{\tan x} \sec^2 x - \frac{5}{x} - \frac{1}{2x} - \frac{1}{\operatorname{Arctan} x}\left(\frac{1}{1+x^2}\right)$

$$\boxed{Y' = \frac{\sin^2 x \, e^{3x} \tan x}{x^5 \sqrt{x} \, \operatorname{Arctan} x} \left[2 \cot x + 3 + \frac{\sec^2 x}{\tan x} - \frac{9}{2x} - \frac{1}{\operatorname{Arctan} x (1+x^2)}\right]}$$

⑥ (a) $Y = \sec^2(1+x^2)$

$$= \frac{1}{\cos^2(1+x^2)}$$

$$= \left[\cos(1+x^2)\right]^{-2}$$

$$Y' = -2\left[\cos(1+x^2)\right]^{-3} \qquad (1+x^2)\, 2x$$

$$\boxed{Y' = \frac{-4x\sin(1+x^2)}{\cos^3(1+x^2)}}$$

(b) $xY^4 + x^2Y = x + 3Y$

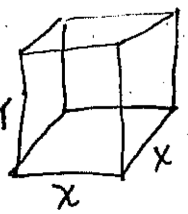
$$Y^4 + x(4Y^3Y') + 2xY + x^2Y' = 1 + 3Y'$$

$$Y'(4xY^3 + x^2 - 3) = 1 - Y^4 - 2xY$$

$$\boxed{Y' = \frac{1 - Y^4 - 2xY}{4xY^3 + x^2 - 3}}$$

(c) $\boxed{f'(x) = \sec^2(\cos(\sqrt{\sin x}))(-\sin\sqrt{\sin x})\left(\tfrac{1}{2}(\sin x)^{-1/2}\right)\cos x}$

⑦



we know $S.A = 337.5 = 2x^2 + 4xY$

so $Y = \dfrac{337.5 - 2x^2}{4x}$

we want to maximize

$$V = x^2 Y$$

$$V = x^2\left(\frac{337.5 - 2x^2}{4x}\right)$$

$$V = \frac{1}{4}\left(337.5x - 2x^3\right)$$

$$V' = \frac{1}{4}\left(337.5 - 6x^2\right)$$

$$V' = 0 \implies 337.5 = 6x^2$$

$$\frac{337.5}{6} = x^2$$

$$x = \pm\sqrt{56.25}$$

$$= \pm 7.5$$

ONLY $x = 7.5$ CM "MAKES SENSE" AS A DIMENSION

WE CHECK IF IT IS A MAXIMUM

INTERVAL	$(-7.5, 7.5)$	$(7.5, \infty)$
TEST	0	8
SIGN OF V'	$+$	$-$
BEHAVIOR OF V	$\nearrow$	$\searrow$

$x = 7.5$ produces MAX VOLUME

$$Y = \frac{337.5 - 2x^2}{4x} = 7.5$$

THE DIMENSIONS ARE $\boxed{7.5\text{ CM} \times 7.5\text{ CM} \times 7.5\text{ CM}}$

8 (a) $x = 0 \quad y = -\frac{1}{2}$

$y = 0$ impossible

(b) $\displaystyle \lim_{x \to \infty} \frac{2x^2}{x^2-4} = \lim_{x \to \infty} \frac{2x^2/x^2}{x^2/x^2 - 4/x^2} = \frac{2}{1} = 2$

$\displaystyle \lim_{x \to -\infty} \frac{2x^2}{x^2-4} = 2$

(c) DOMAIN $\mathbb{R} \setminus \{\pm 2\}$

(d) THERE ARE V.A.'s AT $x = \pm 2$ BECAUSE f DNE there &

$\displaystyle \lim_{x \to -2^-} f(x) \to +\infty$

$\displaystyle \lim_{x \to -2^+} f(x) \to -\infty$

$\displaystyle \lim_{x \to 2^-} f(x) \to -\infty$

$\displaystyle \lim_{x \to 2^+} f(x) \to +\infty$

(e) $f'(x)$ DNE AT $x = \pm 2$

$f'(x) = 0$ AT $x = 0$

INTERVALS	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
test	-3	-1	1	3
SIGN OF f'	$+$	$+$	$-$	$-$
BEHAVIOUR OF f	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$

MAX AT $x = 0$

$y = -\frac{1}{2}$

(f) f'' DNE at ± 2

Intervals	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
test	-3	0	3
sign of f''	$+$	$-$	$+$
concavity	$\cup$	$\cap$	$\cup$

NO INFL. pts