NAME: SOLUTIONS

TEST 1B

DAWSON COLLEGE

NYA-Electrotech Section 7 - Calculus 1

Instructor: E. Richer

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This test is marked out of **65 points**

Question 1. *(4 marks each)*

Find the derivative of each of the following functions. For each function, indicate which of the rules (if any) you have used among: the product rule, quotient rule, chain rule as well as how many times you have used each.

(a) $f(x) = 2(-3x^4 + 2x^2 + \frac{1}{x^2})(\ln x)$

$f(x) = (-6x^4 + 4x^2 + \frac{2}{x^2}) \ln x$ product RULE

$f'(x) = (-24x^3 + 8x - 4x^{-3}) \ln x + \frac{1}{x}(-6x^4 + 4x^2 + \frac{2}{x^2})$

$= \boxed{(-24x^3 + 8x - \frac{4}{x^3}) \ln x + (-6x^3 + 4x + \frac{2}{x^3})}$

(b) $g(t) = \sin(\frac{e^t}{2t-4t^2})$ QUOTIENT RULE (inside) CHAIN RULE

$g'(t) = \boxed{\cos\left(\frac{e^t}{2t-4t^2}\right) \cdot \left(\frac{e^t(2t-4t^2) - (2-8t)e^t}{(2t-4t^2)^2}\right)}$

1 ½ 1 ½

(c) $f(t) = -\frac{1}{6t^6} + \pi^3 - \frac{7t^2}{t^{-8}} + 2\sqrt{t}$

$= -\frac{1}{6}t^{-6} + \pi^3 - 7t^{10} + 2t^{1/2}$

$f'(t) = \boxed{t^{-7} - 70t^9 + t^{-1/2}}$

(d) $h(x) = \cos(3x)\sin(-4x)$

PRODUCT RULE
2 CHAIN RULES

$h'(x) = [-\sin(3x)\cdot 3][\sin(-4x)] + [\cos(-4x)\cdot(-4)][\cos(3x)]$

$= \boxed{-3(\sin(3x))(\sin(-4x)) - (4\cos(-4x)\cos(3x))}$

(e) $g(t) = \sqrt{\ln(5t^2 - t)} = [\ln(5t^2 - t)]^{1/2}$

CHAIN RULE
INSIDE
CHAIN RULE

$\boxed{g'(t) = \frac{1}{2}[\ln(5t^2-t)]^{-1/2}\left[\frac{1}{5t^2-t}\right]\cdot[10t-1]}$

(f) $g(x) = \frac{1}{2}e^{(\tan x)(x^2-2x)}$

product Rule
INside
CHAIN rule

$\boxed{g'(x) = \frac{1}{2}e^{(\tan x)(x^2-2x)} \cdot [\sec^2 x\,(x^2-2x) + (2x-2)\tan x]}$

Question 2. *(6 marks)*
Consider the tangent lines to the curves $y_1 = \frac{1}{4}\sin(\pi e^{x-1})$ and $y_2 = \frac{1}{8\pi}(2x^2+2)^2$ at $x = 1$. Are these tangent lines are parallel, perpendicular or neither?

SLOPES OF THE TANGENT line = DERIVATIVE

$$m_1 = y_1' = \frac{1}{4}\cos(\pi e^{x-1})\cdot(\pi e^{x-1})$$

AT $X=1 \Rightarrow m_1 = \frac{1}{4}(\cos\pi)\cdot\pi$
$= -\frac{\pi}{4}$

$$m_2 = y_2' = \frac{1}{8\pi}2(2x^2+2)\cdot(4x)$$

AT $X=1 \Rightarrow m_2 = \frac{2(4)(4)}{8\pi}$
$= \frac{4}{\pi}$

$$m_1 \cdot m_2 = \left(-\frac{\pi}{4}\right)\left(\frac{4}{\pi}\right)$$
$$= -1$$

THE TANGENT LINES Are $\boxed{\text{perpendicular}}$

Question 3. *(4 marks)*
Find the derivative of $f(x) = -2x\sin x e^x$

$$f'(x) = (-2\sin x + (-2x)\cos x)e^x + e^x(-2x\sin x)$$
$$= \boxed{[-2\sin x - 2x\cos x - 2x\sin x]\,e^x}$$

Question 4. *(12 marks)*
Evaluate the following limits algebraically (not using a table of values). If the limit does not exist; write DNE and determine if the one-sided limits tend to $+\infty$ or $-\infty$ (here you can using a table of values).

(a) *(3 marks)* $\lim_{x\to 9} \frac{3-\sqrt{x}}{x^2-81}$

$$= \lim_{x\to 9} \frac{3-\sqrt{x}}{x^2-81} \frac{(3+\sqrt{x})}{(3+\sqrt{x})}$$

$$= \lim_{x\to 9} \frac{(9-x)}{(x-9)(x+9)(3+\sqrt{x})}$$

$$= \lim_{x\to 9} \frac{-\cancel{(x-9)}}{\cancel{(x-9)}(x+9)(3+\sqrt{x})}$$

$$= \frac{-1}{18(6)} = \boxed{\frac{-1}{108}}$$

(b) *(2.5 marks)* $\lim_{x\to\infty} \frac{6x^4-5x^2+x-5}{2x^4+3x^2-7x}$

$$= \lim_{x\to\infty} \frac{\frac{6x^4}{x^4} - \frac{5x^2}{x^4} + \frac{x}{x^4} - \frac{5}{x^4}}{\frac{2x^4}{x^4} + \frac{3x^2}{x^4} - \frac{7x}{x^4}} = \lim_{x\to\infty} \frac{6 - \cancel{\frac{5}{x^2}}^{\to 0} + \cancel{\frac{1}{x^3}}^{\to 0} - \cancel{\frac{5}{x^4}}^{\to 0}}{2 + \cancel{\frac{3}{x^2}}^{\to 0} - \cancel{\frac{7}{x^3}}^{\to 0}}$$

$$= \frac{6}{2} = \boxed{3}$$

(c) *(2.5 marks)* $\lim_{x\to -2} \frac{2x+4}{x^2-4}$

$$= \lim_{x\to -2} \frac{2\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$$

$$= \frac{2}{-4} = \boxed{\frac{-1}{2}}$$

(d) *(4 marks)* $\lim_{x\to 2}\frac{2x+4}{x^2-4}$

$$= \lim_{x\to 2}\frac{2\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$$

$$= \lim_{x\to 2}\frac{2}{x-2}$$

WE CANNOT SIMPLIFY FURTHER

THE $\boxed{\text{LIMIT DNE}}$

LEFT

x	1.9	1.99	1.999
$\frac{2}{x-2}$	-20	-200	-2000

$\lim_{x\to 2^-}\frac{2x+4}{x^2-4}$ $\boxed{\text{tends TO } -\infty}$

RIGHT

x	2.1	2.01	2.001
$\frac{2}{x-2}$	20	200	2000

$\lim_{x\to 2^+}\frac{2x+4}{x^2-4}$ $\boxed{\text{tends TO } +\infty}$

Question 5. *(5 marks)*
Determine if the function $g(x)$ is continuous at $x=-2$. **Justify** your answer using the 3 conditions necessary for continuity at a point.

$$g(x)=\begin{cases} 3x^2-2x & x<-2 \\ -12x-8 & x>-2 \\ 16 & x=-2 \end{cases}$$

① $g(-2) = 16$

② $\lim_{x\to -2^-} g(x) = \lim_{x\to -2^-} 3x^2-2x$

$= 3(-2)^2-2(-2)$

$= 12+4 = 16$

$\lim_{x\to -2^+} g(x) = \lim_{x\to -2^+} -12x-8$

$= -12(-2)-8$

$= 24-8 = 16$

so $\lim_{x\to -2} g(x) = 16$

③ $\lim_{x\to -2} g(x) = g(-2) = 16$ $\boxed{g(x) \text{ is CONTINUOUS AT } x=-2}$

Question 6. *(5 marks)*
Use the limit definition of the derivative to find the derivative of the function $f(x) = 3x - \frac{1}{x}$. Note that no marks will be given if you do not use the limit definition to evaluate the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h) - \frac{1}{x+h} - 3x + \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \left[3h - \frac{1}{x+h} + \frac{1}{x}\right] \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \left[3h + \frac{-x + x + h}{(x+h)x}\right] \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \left[3h + \frac{h}{(x+h)x}\right] \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} 3 + \frac{1}{(x+h)x}$$

$$= \boxed{3 + \frac{1}{x^2}}$$

Question 7. *(4 marks)*
Find the value of the constant a if the slope of the tangent line to the curve $y = -2ax^2 + 2x + 2$ at $x = 1$ is equal to 8.

$$y' = -4ax + 2$$

AT $x = 1$ $\quad y' = -4a(1) + 2$
$\qquad\qquad\qquad = -4a + 2$

AT $x = 1$ $\quad y' = 8 \implies$

$$8 = -4a + 2$$
$$6 = -4a$$
$$\boxed{a = -3/2}$$

Question 8. *(5 marks)*
The distance s (in metres) traveled by a metro train after the breaks are applied is given by $s = 12t - 2t^2$, where t is the time (in seconds).
(a) How far has the train traveled 2 seconds after the breaks are applied?
(b) How much time elapses between the moment the brakes are applied and the moment the train stops?
(c) How far has the train traveled between applying the brakes and coming to a stop?

(a) $s(2) = 12(2) - 2(2)^2$
$= 24 - 8$
$= \boxed{16\text{ M}}$

(b) WHEN THE TRAIN STOPS velocity $= 0$

$v(t) = s'(t) = 12 - 4t$

$v(t) = 0$ when $12 - 4t = 0$
$12 = 4t$
$\boxed{t = 3\text{ s}}$

(c) $S(3) = 12(3) - 2(3)^2$
$= 36 - 18$
$= \boxed{18\text{M}}$