

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 10 (A)

Question 1. (6 marks)

(a) Find the formula for the general term a_n of the sequence assuming the pattern of the few terms continues:

$$\left\{ -\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \dots \right\}$$

$$a_n = \frac{(-1)^n \cdot n}{(n+1)^2}$$

(b) Determine if the sequence $\{a_n\}$ converges where

$$\begin{aligned} a_n &= \frac{(n+1)!}{(n+3)!} = \frac{1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-2)(n-1)(n)(n+1)}{1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-2)(n-1)(n)(n+1)(n+2)(n+3)} \\ &= \frac{1}{(n+2)(n+3)} \end{aligned}$$

$$\therefore \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{(n+2)(n+3)} = 0 \quad (\text{converges})$$

Question 2. (4 marks) Find the sum of the series if it converges:

$$\sum_{n=2}^{\infty} \frac{3}{5^n} = \sum_{n=2}^{\infty} \frac{3}{5 \cdot 5^{n-1}} = \sum_{n=2}^{\infty} \frac{3}{5} \cdot \left(\frac{1}{5}\right)^{n-1}$$

$$= \sum_{n=2}^{\infty} \frac{3}{5} \left(\frac{1}{5}\right)^{n-1} + \frac{3}{5} \left(\frac{1}{5}\right)^0 - \frac{3}{5} \left(\frac{1}{5}\right)^0$$

$$= \sum_{n=1}^{\infty} \frac{3}{5} \left(\frac{1}{5}\right)^{n-1} - \frac{3}{5} = \frac{3/5}{1 - 1/5} - \frac{3}{5}$$

$$= \frac{3}{4} - \frac{3}{5} = \frac{3}{20}$$

GEOMETRIC:
 $a = \frac{3}{5}$ $r = \frac{1}{5}$