Dawson College: Calculus II: 201-NYB-05 05

April 21, 2011

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 10 (B)

Question 1. *(6 marks)*

(a) Determine if the sequence $\{a_n\}$ converges where

$$a_n = \frac{(n+3)!}{(n+1)!} = \frac{1\cdot 2\cdot 3\cdot \ldots \cdot (n-1)(n)(n+1)(n+2)(n+3)}{1\cdot 2\cdot 3\cdot \ldots \cdot (n-1)(n)(n+1)}$$

$$= (n+2)(n+3)$$

$$\therefore \lim_{n\to\infty} a_n = \lim_{n\to\infty} (n+2)(n+3) = \infty \quad \therefore \text{ THE SEQUENCE DIVERGES.}$$

(b) Find the formula for the general term a_n of the sequence assuming the pattern of the few terms continues:

$$\left\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \cdots\right\}$$

$$a_n = \frac{(-1)^n \cdot n}{(n+1)^2}$$

Question 2. *(4 marks)* Find the sum of the series if it converges:

$$\sum_{n=2}^{\infty} \frac{4}{7^n} = \sum_{n=2}^{\infty} 4\cdot\left(\frac{1}{7}\right)^n = \sum_{n=2}^{\infty} \frac{4}{7}\cdot\left(\frac{1}{7}\right)^{n-1}$$

$$= \sum_{n=2}^{\infty} \frac{4}{7}\cdot\left(\frac{1}{7}\right)^{n-1} + \frac{4}{7}\left(\frac{1}{7}\right)^0 - \frac{4}{7}\left(\frac{1}{7}\right)^0$$

$$= \sum_{n=1}^{\infty} \frac{4}{7}\left(\frac{1}{7}\right)^{n-1} - \frac{4}{7} = \frac{a}{1-r} - \frac{4}{7}$$

GEOMETRIC

$a = \frac{4}{7}$

$r = \frac{1}{7}$

$$= \frac{\frac{4}{7}}{1-\frac{1}{7}} - \frac{4}{7} = \frac{4}{7}\cdot\frac{7}{6} - \frac{4}{7} = \frac{2}{3} - \frac{4}{7} = \frac{2}{21}$$